2.2 General Theory of Relativity of Field-Space-Mechanics

- *Draft version* -

The purpose of this chapter is to define the field equations based on the geometric foundations of the FSM model. The predicted and heuristically derived relationships presented in the following chapters are thus scientifically valid. Simulation models can be developed on this basis. For specific applications, the general formulas and relationships must be further refined and defined. These elaborated mathematical foundations form the basis for establishing further relationships. For the sake of clarity, we intend to remain within these foundations for the purposes of this paper.

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1. Precise diagonal metric

The FSM predicts a dynamic 7-dimensional field-space with time ($D_{1\text{-}3}$ as visible dimensions in the particle-field $F_{1\text{-}3}$; $D_{4\text{-}6}$ as compact dimensions in the wave-field $F_{4\text{-}6}$, time t). The observable space in the particle-field $F_{1\text{-}3}$ is understood as a projection of the wave-field $F_{4\text{-}6}$ or as a hologram. The metric must be able to quantify the relationships described in the axioms (**Chapter 1.2**). It is diagonal to ensure the orthogonality of the dimensional planes D_{45}, D_{46}, D_{56}, and contains scaling factors for possible photon subspaces.

Definition of dimensions and indices:

- 7-dimensional (7D) space-time indices with field-space and time; Lorentz indices, covariant at the bottom, contravariant at the top with metric
- M, N = 0, 1, 2, 3, 4, 5, 6
- 0 – time coordinate (t), unit : m, normalised by the product (ct)
- 1, 2, 3 – visible space-time dimensions of the 'particle-field'; x, y, z; unit: m
- 4, 5, 6 – compact dimensions of the 'wave-field'; y^4, y^5, y^6; unit: m; this is the space for internal degrees of freedom that generate wave fields
- μ, ν = 0, 1, 2, 3 – only the visible 4D space-time indices in the particle-field
- m, n = 4, 5, 6 – only the compact 3D wave-field dimensions

Global and local gravitational effects dominate in visible 4D space. Compact space generates massive fields through reduction.

Starting point – Minkowski metric with the STR metric in 4D:

$$-\frac{dx^2}{dt^2} - \frac{dy^2}{dt^2} - \frac{dz^2}{dt^2} + c^2 = \frac{ds^2}{dt^2}$$

$$ds^2 = -c^2\, dt^2 + dx^2 + dy^2 + dz^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu \qquad (2.10)$$

- ds^2 – the invariant length square (interval); unit: m²; measures the space-time distance
- $\eta_{\mu\nu}$ – diag(-1,1,1,1) the metric tensor (flat, Lorentz-signed); dimensionless
- c – maximum speed $V_{max} = c = 299792458\ \frac{\text{m}}{\text{s}}$

Extension to 7D – Minkowski metric with the STR metric in 7D::

Physical space-time is a smooth, orientable, pseudo-Riemannian manifold. The FSM predicts that the wave-field $F_{4\text{-}6}$ is linked to the visible particle-field $F_{1\text{-}3}$ via its compact dimensions.

$7D = 4D_x \times 3D_y$

$$ds^2 = -c^2\, dt^2 + dx^2 + dy^2 + dz^2 + dy_4^2 + dy_5^2 + dy_6^2 = \eta_{MN}\, dx^M\, dx^N \qquad (2.11)$$

- η_{MN} – Diagonal metric; signature (-, +, +, +, +, +, +)

The diagonal metric describes an empty, flat 7-dimensional space-time without gravity or oscillations.

Introduction of a local metrical disturbance h_{MN}:

To take into account gravity, vector and scalar fields, the perturbation h_{MN} is added:

$$g_{MN} = \eta_{MN} + h_{MN} \tag{2.12}$$

- h_{MN} – perturbative disturbance; dimensionless

Block structure for h_{MN}:

$$h_{MN} = \begin{pmatrix} h_{\mu}v & h_{\mu m} \\ h_{m}v & h_{mn} \end{pmatrix} \tag{2.13}$$

a) $h_{\mu\nu}$ – Gravity with an oscillating isotropic gravitational disturbance

The 4D-visible curvature must take into account the periodic cavity vibration in the form of a mathematical rotation. **Figures 2.11** and **2.12** show that the local electromagnetic oscillation, as a subspace within the universe, merely permits a periodic variation in its inertial force between cos(kt = 0) = 1 for the maximum at the location of the D_{56} dimensional plane and cos(kt = 90°) = 0.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{2.14}$$

Static gravity generates the Schwarzschild metric:

$$h_{\mu\nu} = \frac{2\,G\,M}{c^2\,r_S}\,\delta_{\mu\nu}$$

A periodic disturbance term counteracting the static gravitational force is now taken into account. This results in an oscillating isotropic gravitational disturbance:

$$h_{\mu\nu} = \frac{G\,M}{c^2\,r}\,(1+\cos(kt+\beta))\,\delta_{\mu\nu} \tag{2.15}$$

$$h_{00} = 2\,\frac{G\,M}{c^2\,r}$$ → Reduction to a static solution according to Schwarzschild

In component notation:

$$h_{tt} = -\frac{G\,M}{c^2\,r}\,(1 + \cos(kt + \beta));\; h_{xx} = h_{yy} = h_{zz} = \frac{G\,M}{c^2\,r}\,(1 + \cos(kt + \beta))$$

$h_{\mu\nu} = 0$ for $\mu \neq \nu$

- G – gravitational constant; $G = 6{,}674\ 10^{-11}\ \frac{\text{m}^3}{\text{kg s}^2}$
- M – mass; unit: kg
- c – maximum speed $V_{max} = c = 299792458\ \frac{\text{m}}{\text{s}}$
- r – field radius; unit: m
- k – angular frequency; unit: $\frac{1}{\text{s}}$
- $(1+ \cos(kt))$ – oscillation of the local disturbance; describes the periodic evolution of inertia parallel to the fourth dimension; it takes into account the relationship: $c^2 = V_4^2 + V_5^2$
  a) $\cos(kt = 0) = 1$ – point of contact parallel to the plane of dimensions D_{56}; maximum amplitude of the inertial motion
  b) $\cos(kt = 90°) = 0$ means an orthogonal displacement of its inertial force relative to the dimension plane D_{56}, smallest amplitude of inertial motion
  c) For objects in a state of rotation, the intrinsic rotation that averages out the oscillation results in $[1 + \cos(kt) = 1]$ with $\frac{G\,M}{c^2\,r}$
  d) For the static case, this yields the fixed maximum, the Schwarzschild dynamics $\frac{2\,G\,M}{c^2\,r}$
- β – deviation angle as fixed phase shift; unit: rad, dimensionless; $0° < \beta < 90°$; explains coordinate-dependent interaction effects
  a) At 0°, the field exchange with the particle field is maximally parallel to the dimensional plane D_{56}. → strong interaction dominates completely
  b) A possible deviation of β up to 90° results in a shift from strong interaction to weak interaction
  c) Although the interaction for $\beta = 90°$ persists in the wave-field, it no longer acts in the particle-field
  d) Explains the possible interaction of dark matter with 4D
- $\delta_{\mu\nu}$ – Kronecker delta; 1 at $\mu = \nu$; 0 otherwise

b) $h_{\mu m}$ – perturbable vector fields

$$h_{\mu m} = h_{m\mu} = (A^a_\mu + \delta A^a_\mu)\,\delta_{am}$$

- Indices: a, m = 4, 5, 6; μ = 0, 1, 2, 3
- A^a_μ – Vector field in 7D; geometrically, it links the wave-field to the particle-field; following compactification to 4D, it behaves as a gauge potential.

- δA_μ^a – Possibility of deviation, e.g. due to an external disturbance via the particle-field; compensating forces counteract the disturbance.
- δ_{am} – dimensional planes

c) $h_{m\nu}$ – perturbable vector fields

$h_{m\nu} = h_{\nu m} = (A_\nu^a + \delta A_\nu^a)\ \delta_{am}$

- Indices: a, m = 4, 5, 6; ν = 0, 1, 2, 3

d) h_{mn} – Curvature in the wave-field

$h_{mn} = 0$

- In the wave-field, there is no curvature of space-time, but only vector fields which, within this Euclidean space ($3D_y$), model the deformation of space-time in particle-field ($4D_x$).
- Following compactification, scalar terms will appear at this point that stabilise a dynamic space-time deformation via feedback.

Incorporation of the global curvature of the universe:

The universe possesses its own inertial motion in space-time, which further influences the surrounding curvature of objects. The effect of global curvature is measured by the deviation from orthogonality to the D_{56} dimensional plane for all objects. This additional space-time curvature describes the influence of dark energy on massive objects. For g_{MN}, the term

$[1 + \cos(k_{Uni}\, t)]$

is introduced to account for the global influence of dark energy on local space-time curvature. This term decreases dynamically to a factor of (1 + cos(90°)) as the universe expands to its maximum size.

$$g_{MN} = [1 + \cos(k_{Uni}\, t)]\ (\eta_{MN} + h_{MN}) \qquad (2.16)$$

Explicit matrix: (2.17)

$$g_{MN} = \begin{pmatrix} g_{00} & 0 & 0 & 0 & A_0^4 + \delta A_0^4 & A_0^5 + \delta A_0^5 & A_0^6 + \delta A_0^6 \\ 0 & g_{11} & 0 & 0 & A_1^4 + \delta A_1^4 & A_1^5 + \delta A_1^5 & A_1^6 + \delta A_1^6 \\ 0 & 0 & g_{22} & 0 & A_2^4 + \delta A_2^4 & A_2^5 + \delta A_2^5 & A_2^6 + \delta A_2^6 \\ 0 & 0 & 0 & g_{33} & A_3^4 + \delta A_3^4 & A_3^5 + \delta A_3^5 & A_3^6 + \delta A_3^6 \\ A_0^4 + \delta A_0^4 & A_1^4 + \delta A_1^4 & A_2^4 + \delta A_2^4 & A_3^4 + \delta A_3^4 & g_{44} & 0 & 0 \\ A_0^5 + \delta A_0^5 & A_1^5 + \delta A_1^5 & A_2^5 + \delta A_2^5 & A_3^5 + \delta A_3^5 & 0 & g_{55} & 0 \\ A_0^6 + \delta A_0^6 & A_1^6 + \delta A_1^6 & A_2^6 + \delta A_2^6 & A_3^6 + \delta A_3^6 & 0 & 0 & g_{66} \end{pmatrix}$$

$g_{00} = - c^2 [1 + \cos(k_{Uni}\, t)] \left(1 - \frac{G\,M}{c^2\, r} (1 + \cos(kt + \beta))\right)$

$g_{11} = g_{22} = g_{33} = [1 + \cos(k_{Uni}\, t)] \left(1 + \frac{G\,M}{c^2\, r} (1 + \cos(kt + \beta))\right)$

$g_{44} = g_{55} = g_{66} = [1 + \cos(k_{Uni}\, t)]$

Line element ds^2:

$ds^2 = g_{MN}\, dx^M\, dx^N$

$ds^2 = [1 + \cos(k_{Uni}\, t)] \times$ [flat 7D Minkowski space + oscillating isotropic gravitational perturbation + vector coupling terms]

$$ds^2 = [1 + \cos(k_{Uni}\, t)] \left[- c^2\, dt^2 + dx^2 + dy^2 + dz^2 + dy_4^2 + dy_5^2 + dy_6^2 + \frac{G\,M}{c^2\, r} (1 + \cos(kt + \beta)) (c^2\, dt^2 + dx^2 + dy^2 + dz^2) + 2(A_\mu^a + \delta A_\mu^a)\, dy_a\, dx^\mu\right] \quad (2.18)$$

- $[1 + \cos(k_{Uni}\, t)]$ – dilatational cosmic oscillation; k_{Uni} – the angular frequency of the universe; a factor of 1 results in the observable relativity of space-time; the dynamics $\cos(k_{Uni}\, t)$ result in a global, non-observable relativity, explaining the ratio between dark energy and already coupled matter
- $[- c^2\, dt^2 + dx^2 + dy^2 + dz^2]$ – flat 4D space-time as a background; STR
- $[dy_4^2 + dy_5^2 + dy_6^2]$ – flat, compact wave-field dimensions; similar to Kalzua-Klein
- $[\frac{G\,M}{c^2\, r} (1 + \cos(kt + \beta)) (c^2\, dt^2 + dx^2 + dy^2 + dz^2)]$ – oscillating isotropic gravity; similar to Schwarzschild, but with oscillating inertial dynamics for arbitrarily scalable electromagnetic waves
  $(1 + \cos(kt + \beta))$ – focus on amplitude → factor 2 (Schwarzschild solution)
  $(1 + \cos(kt + \beta))$ – the dynamic case produces factor 1 (Kerr-like)
- β – for the D_{56} dimensional plane: maximum interaction at $\beta = 0$, dynamically weak interaction at $0 < \beta < 90°$
- $[2(A_\mu^a + \delta A_\mu^a)\, dy_a\, dx^\mu]$ – perturbable vector fields from wave-field dimensions; Kaluza–Klein-like
- A_μ^a – Vector field in 7D; dimensionless in the 7D context; with compactification, the effective 4D coupling is achieved through rescaling: $A_\mu^a \rightarrow A_\mu^a / R$

The metric implements the definition given in **Chapter 1.1**.

Reduction to STR/GTR and verification:

The first step is to eliminate the compact wave-field dimensions for the observable 4D space-time (t, x, y, z). Consequently, the movements:

$dy_4^2 = dy_5^2 = dy_6^2 = 0$

$ds^{2\,(4)} = [1 + \cos(k_{Uni}\,t)]\,[- c^2\,dt^2 + dx^2 + dy^2 + dz^2 + \frac{G\,M}{c^2\,r}\,(1 + \cos(kt + \beta))\,(c^2\,dt^2 + dx^2 + dy^2 + dz^2]$

$ds^{2\,(4)} = [1 + \cos(k_{Uni}\,t)]\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + h_{\mu\nu}\,dx^\mu\,dx^\nu$

GTR and STR ignore the global curvature $\cos(k_{Uni}\,t)$ in the universe:

$\cos(k_{Uni}\,t) = 0$

$ds^{2\,(4)} = \eta_{\mu\nu}\,dx^\mu\,dx^\nu + h_{\mu\nu}\,dx^\mu\,dx^\nu$

Borderline case STR:

$h_{\mu\nu} \rightarrow 0$

$ds^2 = - c^2\,dt^2 + dx^2 + dy^2 + dz^2$

Borderline case GTR:

a) Fixing the local oscillation at the point of maximum value using $kt = \beta = 0$:

$\cos(kt = 0°) = 1 \rightarrow 1 + \cos(kt = 0°) = 2$

The Schwarzschild solution for the perturbation is as follows::

$$h_{\mu\nu} = \frac{G\,M}{c^2\,r}\,2\,\delta_{\mu\nu}$$

Using the static approximation, this leads to the Schwarzschild metric.

b) Oscillation about the time-averaged value:

$\langle\cos(kt) = 0\rangle \rightarrow \langle 1 + \cos(kt) = 1\rangle$ for: $\beta = 0$

The following applies to the effective disturbance during oscillation:

$$h_{\mu\nu} = \frac{G\,M}{c^2\,r}\,\delta_{\mu\nu}$$

This results in half the Schwarzschild strength for models involving rotating objects, as is also the case in the Kerr metric.

<u>An example based on the gravitational lensing effect:</u>

The gravitational lensing effect is a phenomenon in which light rays are deflected by the curvature of space-time around a massive object, resulting in distortions, multiple images, or magnifications of distant sources. The critical impact parameter *b* is derived using both the Schwarzschild and Kerr metrics. To verify the FSM model, the critical impact parameter *b* should also be derived.

Derivation via FSM:

If ignoring the global component, photons are always null-geodesic with:

$ds^2 = 0$

It contains two conserved quantities that are not affected by any external forces:

a) Energy parameter E → internal energy increases with the addition of matter

b) Angular momentum parameter L → accumulates as a result of the absorption of matter the angular momentum

$$ds^2 = 0 = g_{00} \left(\frac{dt}{d\lambda}\right)^2 + g_{rr} \left(\frac{dr}{d\lambda}\right)^2 + g_{\varphi\varphi} \left(\frac{d\varphi}{d\lambda}\right)^2$$

$$\frac{dt}{d\lambda} = -\frac{E}{g_{00}} \; ; \; \frac{d\varphi}{d\lambda} = \frac{L}{g_{\varphi\varphi}}$$

- φ – azimuth angle

Insert into $ds^2 = 0$:

$$0 = g_{00} \left(-\frac{E}{g_{00}}\right)^2 + g_{rr} \left(\frac{dr}{d\lambda}\right)^2 + g_{\varphi\varphi} \left(\frac{L}{g_{\varphi\varphi}}\right)^2$$

With:

$$g_{\varphi\varphi} \approx r^2 \; ; \qquad g_{rr} \approx 1 \; ; \qquad g_{00} = -c^2 \left[1 + \cos(k_{Uni}\, t)\right] \left(1 - \frac{G M}{c^2 r} (1 + \cos(kt + \beta))\right)$$

- $[1 + \cos(k_{Uni}\, t)]$ – omitted with $\cos(k_{Uni}\, t) = 0$ for local viewing
- $(1 + \cos(kt + \beta))$ – generated by g_{00} and simplified to $f = (1 + \cos(kt + \beta))$
- $\frac{1}{g_{00}} = \frac{1}{c^2} \left(1 + \frac{f G M}{c^2 r}\right)$

This leads to the radial equation derived from the null geodesic condition:

$$\left(\frac{dr}{d\lambda}\right)^2 = E^2 \frac{1}{c^2} \left(1 + \frac{f G M}{c^2 r}\right) - \frac{L^2}{r^2}$$

Conversion using $\frac{E^2 r^2}{L^2} = 1$, with c as the natural unit according to convention:

$$\left(\frac{dr}{d\lambda}\right)^2 = E^2 - \left[\frac{L^2}{r^2} \left(1 - \frac{f G M}{c^2 r}\right)\right] = E^2 - V_{eff}(r)$$

The effective potential $V_{eff}(r)$ is factored by the centrifugal term:

$$V_{eff}(r) = \frac{L^2}{r^2} \left(1 - \frac{f G M}{c^2 r}\right)$$

The following is the derivation of the impact parameter b_{crit} as the transition radius for photons that fall into the black hole or are merely deflected. At the photon sphere with radius r_{ph}, a circular orbit holds $\frac{\mathrm{d}r_{ph}}{\mathrm{d}\lambda} = 0$.

$$E^2 - V_{eff}(r) = 0$$

Possible instability occurs as a function of the field radius r_{ph} at $V_{eff}(r)$ auf. To this end, the extremum is determined:

$$\frac{\mathrm{d}V_{eff}}{\mathrm{d}r_{ph}} = \frac{1}{\mathrm{d}r_{ph}} \left[L^2\left(- 2r_{ph}^{-2} - \frac{f\,G\,M}{c^2}\, r_{ph}^{-3}\right)\right] = 0$$

$$\frac{\mathrm{d}V_{eff}}{\mathrm{d}r_{ph}} = L^2 \left(- 2r_{ph}^{-3} + 3\frac{f\,G\,M}{c^2}\, r_{ph}^{-4}\right) = 0$$

$$r_{ph} = 1{,}5\,\frac{f\,G\,M}{c^2} = 1{,}5\; r\, f \rightarrow \text{This is the unstable circular path of a photon.} \qquad (2.19)$$

Using r_{ph} with $r_{ph} = 1{,}5\; r\, f$:

$$E^2 = \frac{L^2}{r_{ph}^{\,2}}\left(1 - \frac{f\,G\,M}{c^2\, r_{ph}}\right) = \frac{L^2}{r_{ph}^{\,2}}\left(1 - \frac{f\,r}{r_{ph}}\right)$$

$$\frac{L^2}{E^2} = 6{,}75\; r^2\, f^2$$

The critical impact parameter b_{crit} is defined as $\frac{L}{E}$:

$$\frac{L}{E} = b_{crit} = \sqrt{6{,}75}\; r\, f = 2{,}598\; r\, f \qquad (2.20)$$

- r – field radius
- $f = (1 + \cos(kt + \beta))$
- β – deviation angle; here, it describes the prograde/retrograde deviation from the orthogonal angle of incidence into the black hole

a) Borderline case:

$$r_{ph} = 1{,}5\; r\,(1 + \cos(kt + \beta))$$

A reference point must be defined as the initial position. This should be defined by an orthogonal angle of incidence at:

$$kt + \beta = 0°$$

$$r_{ph} = 1{,}5\; r\,(1+ 1) = 3\; r$$

$b_{crit} = 2{,}598\; r\, f = 2{,}598\; r\,(1+ 1) = 5{,}196\; r$ → as in the case of Schwarzschild and Kerr

- For b_{crit} < 5,196 r photons are captured by the black hole
- For b_{crit} = 5,196 r photons orbit in an unstable manner on the photosphere
- For b_{crit} > 5,196 r photons are deflected (**gravitational lensing effect**)

The natural limits of the extreme values for prograde and retrograde rotation reach certain maximum values that are multiples of the field radius r.

b) Case for the prograde (with the) rotation:

The minimum value for r_{ph} can only be the simple factor for the field radius r, because the photon can no longer be detected after that.

For: $r_{ph} = r$

$r = 1{,}5\, r\, (1 + \cos(kt = 0° + \beta))$

$\frac{2}{3} = (1 + \cos(kt = 0° + \beta))$

Deviation from the relative value kt = 0°:

$\cos(kt = 0° + \beta)) = \frac{2}{3} - 1 = -\frac{1}{3}$

$\beta = \arccos(-\frac{1}{3}) \approx 109{,}5°$

$b_{crit} = 2{,}598\, (1 + \cos(kt = 0° + \beta = 109{,}5°))$

$b_{crit} = 1{,}73\, r$ (near Kerr)

For: $r_{ph} = 1{,}5\, r$

$1{,}5\, r = 1{,}5\, r\, (1 + \cos(kt = 0° + \beta))$

$1 = (1 + \cos(kt = 0° + \beta))$

$\cos(kt = 0° + \beta)) = 0$

$\beta = \arccos(0) \approx 90°$

$b_{crit} = 2{,}598\, (1 + \cos(kt = 0° + \beta = 90°))$

$b_{crit} = 2{,}598\, r$

With an angle of incidence of < 90° in prograde rotation, a photon is captured by the black hole at a distance of r_{ph} =1,5 r from the photon sphere. The angle of incidence of 109.5° at $r_{ph} = r$ means that a photon moves away from the black hole at an angle of 19.5°. This could apply to matter that, due to electrostatic repulsion, makes an additional negative contribution to the gravitational force.

Hypothetically, the term (1 + cos(kt = 0° + β)) would allow for an amplification up to $r_{ph} = \frac{\lambda_{BH}}{2\pi}$ with β close to 180°. An angle of 180° would correspond to the direct direction out of the black hole.

- λ_{BH} – wavelength of the black hole within the event horizon r

c) Case of retrograde (opposite) rotation:

For: $r_{ph} = 4\, r$

The cosine function is restricted to values between 0 and 1. If the transition is to be considered relative to the critical boundary, then the orthogonal angle to the black hole must be at cos(0) with respect to the opposite quadrant. Turning the quadrant 90° relative to the critical boundary for r_{ph} triggers the necessary reflection. This is accounted for by an additional factor of 1.

$4\,r = 1{,}5\,r\,(1 + 1 + \cos(kt = 0° + \beta))$

$\frac{8}{3} = (1 + 1 + \cos(kt = 0° + \beta)$

$\beta = \arccos(\frac{2}{3}) \approx 48{,}2°$

$b_{crit} = 2{,}598\,((1 + 1 + \cos(kt = 0° + \beta = 48{,}2°)) = 6{,}93\,r$ (near Kerr)

<u>For: $r_{ph} = 4{,}5\,r$</u>

$4{,}5\,r = 1{,}5\,r\,(1 + 1 + \cos(kt = 0° + \beta))$

$3 = (1 + 1 + \cos(kt = 0° + \beta))$

$\beta = \arccos(1) = 0°$ → trigonometric limit

$b_{crit} = 2{,}598\,((1 + 1 + \cos(kt = 0° + \beta = 0°)) = 7{,}794\,r$

At a retrograde angle of incidence of 48.2°, a photon is deflected by the photon sphere at $r_{ph} = 4\,r$. In the case of a 90° angle, deflection occurs already at a distance of $r_{ph} = 4{,}5\,r$.

2. Derivation of the geodesic equations

The geodesic equations define the trajectories of particles and fields in the 7-dimensional FSM geometry. Geodesics are the generalised ‘straight lines’ in curved spaces and explain in the FSM how mass arises from frequency multiples and charge from rotation in compact dimensions. Geodesics of the FSM are comparatively simple because they use the orthogonal dimensions to directly derive causal effects such as gravity as a counterforce to electromagnetism, thus representing the visible particle-field $F_{1\text{-}3}$ as a holographic projection.

<u>Starting point – The invariant line length in the FSM metric:</u>

$$ds^2 = g_{MN}\,dX^M\,dX^N \quad (2.21)$$

- ds^2 – invariant line length square
- g_{MN} – 7-dimensional metric tensor (dimensionless) for curvature and rotations; M, N = 0,…, 6 (0 = time t, 1, 2, 3 = particle-field, 4, 5, 6 = wave-field)
- dX^M bzw. dX^N – infinitesimal coordinate shift; unit: m; to parameterise trajectories

Effect function for geodesists:

Geodesics are the paths that maximise the integral over the arc length (or, equivalently, the proper time τ).

Functional S for geodesists (shortest curves) in 7D:

$$S = \int \mathrm{d}s = \int_{\lambda_1}^{\lambda_2} \sqrt{g_{MN} \frac{\mathrm{d}x^M}{\mathrm{d}\lambda} \frac{\mathrm{d}x^N}{\mathrm{d}\lambda}}\, \mathrm{d}\lambda \qquad (2.22)$$

Zero-geodetic curves or timelike curves:

$$S = - \int_{\lambda_1}^{\lambda_2} \sqrt{- g_{MN} \frac{\mathrm{d}x^M}{\mathrm{d}\lambda} \frac{\mathrm{d}x^N}{\mathrm{d}\lambda}}\, \mathrm{d}\lambda \qquad (2.23)$$

- S – effect/action; unit: m; line length is minimised; $\delta S = 0$ yields equations of motion
- λ – affine parameter; $\lambda = 0, 1, 2, 3, 4, 5, 6$; parameterised trajectory $x^M(\lambda)$; is orthogonal to the velocity; proportional to the proper time τ; unit normalised to: m
- τ – proper time; $\tau = t_{Obj}$; unit: s; causes for solid particles: $\mathrm{d}s^2 = - c^2\, \mathrm{d}\tau^2$
- $g_{MN}(M)$ – 7-dimensional metric tensor; dynamic with $\cos(kt)$; defines the 'length' in curved space-time
- $\frac{\mathrm{d}X^M}{\mathrm{d}\lambda}$ or $\frac{\mathrm{d}X^N}{\mathrm{d}\lambda}$ – speed along the trajectory (tangential vector); describes the direction and speed of movement; unit: $\frac{\mathrm{m}}{\mathrm{m}}$ becomes dimensionless

For massive particles, the additional condition $g_{MN}\, \dot{X}^M \dot{X}^N = - c^2$ as a normalised tangential vector also applies, but the direct variation of S already leads to the affine geodesic.

The Lagrange function (Lagrangian):

The integrand of the impact function is the Lagrange function $\mathcal{L}$:

$$\mathcal{L} = \frac{\mathrm{d}s}{\mathrm{d}\lambda}$$

For timelike:

$$\mathcal{L} = - \sqrt{- g_{MN} \frac{\mathrm{d}x^M}{\mathrm{d}\lambda} \frac{\mathrm{d}x^N}{\mathrm{d}\lambda}} = -\sqrt{- g_{MN}\, \dot{x}^M \dot{x}^N} \qquad \text{with: } \dot{x}^M = \frac{\mathrm{d}x^M}{\mathrm{d}\lambda}$$

Or, equivalently, for the extreme case:

$$\mathcal{L}(X, \dot{X}) = -\frac{1}{2}\, g_{MN}(X)\, \dot{x}^M \dot{x}^N \qquad (2.24)$$

- $\mathcal{L}$ – is scalar density; is a kinetic energy-like function for geodesics; unit: $\frac{m^2}{m^2}$; becomes dimensionless

<u>The Euler-Lagrange equations:</u>

For a system with coordinates x^λ (λ = 0,…, 6), the Euler-Lagrange equations are:

$$\frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^\lambda}\right) - \frac{\partial \mathcal{L}}{\partial x^\lambda} = 0 \qquad (2.25)$$

- $\frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^\lambda}\right)$ – term minimises the action S and defines geodesics; unit: $\frac{1}{m}$
- $\frac{\partial \mathcal{L}}{\partial \dot{x}^\lambda}$ – canonical momentum (covariant) with respect to the coordinate x^λ; similar to the classic pulse $p = \frac{\partial \mathcal{L}}{\partial V}$; unit: $\frac{m}{m}$ becomes dimensionless
- $\frac{\partial \mathcal{L}}{\partial x^\lambda}$ – time derivation; explicit dependence on coordinates via $g_{MN}(x)$ generates 'forces' through curvature; unit: $\frac{1}{m}$

<u>Calculation of the derivatives:</u>

Impulse p_λ:

$$p_\lambda = \frac{\partial \mathcal{L}}{\partial \dot{x}^\lambda} = \frac{1}{2} \cdot 2\, g_{KN}\, \dot{x}^N = g_{KN}\, \dot{x}^N$$

Time derivative of the impulse:

$$\frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^\lambda}\right) = \frac{d}{d\lambda}\left(g_{\lambda N}\, \dot{x}^N\right) = \partial_M g_{\lambda N}\, \dot{x}^M \dot{x}^N + g_{\lambda N}\, \ddot{x}^N$$

- $\partial_M g_{\lambda N}$ – spatio-temporal derivation of the metric; unit: $\frac{1}{m}$
- $\dot{x}^M$ or $\dot{x}^N$ – unit: $\frac{m}{m}$ becomes dimensionless
- $\ddot{x}^N = \frac{d^2 x^N}{d\lambda^2}$ – unit: $\frac{1}{m}$

Explicit derivation:

$$\frac{\partial \mathcal{L}}{\partial x^\lambda} = \frac{1}{2}\left(\partial_\lambda g_{MN}\right) \dot{x}^M \dot{x}^N$$

- $(\partial_\lambda g_{MN})$ – is the derivative of the metric with respect to x^λ; unit: $\frac{1}{\text{m}}$

Insert into the Euler-Lagrange equation:

$$\frac{d}{d\lambda}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}^\lambda}\right) - \frac{\partial\mathcal{L}}{\partial x^\lambda} = 0 \rightarrow \partial_M g_{\lambda N}\,\dot{x}^M\,\dot{x}^N + g_{\lambda N}\,\ddot{x}^N - \frac{1}{2}\,(\partial_\lambda g_{MN})\,\dot{x}^M\,\dot{x}^N = 0$$

Multiplying by -1 and rearranging gives:

$$- g_{\lambda N}\,\ddot{x}^N - \partial_M g_{\lambda N}\,\dot{x}^M\,\dot{x}^N + \frac{1}{2}\,(\partial_\lambda g_{MN})\,\dot{x}^M\,\dot{x}^N = 0$$

$$\ddot{x}^\lambda = g^{\lambda P}\,(\partial_M g_{PN}\,\dot{x}^M\,\dot{x}^N - \frac{1}{2}\,(\partial_P g_{MN})\,\dot{x}^M\,\dot{x}^N) \qquad (2.26)$$

- $g^{\lambda P}$ – inverse metric; dimensionless
- Index: P = 0, 1, 2, 3, 4, 5, 6; contravariant

Transformation to the geodesic equation with Christoffel symbols:

The standard form of Christoffel symbols of the second kind is:

$$\Gamma^\lambda_{MN} = \frac{1}{2}\,g^{\lambda P}\,(\partial_M g_{NP} + \partial_N g_{MP} - \partial_P g_{MN})$$

- Γ^λ_{MN} – Christoffel's symbol; unit: $\frac{1}{\text{m}}$; describes the curvature/affine connection → shows the extent to which the curvature depends on time-deforming massive objects

By inserting and renaming the indices, it can be seen that the expression is exactly:

$$\ddot{x}^\lambda + \Gamma^\lambda_{MN}\,\dot{x}^M\,\dot{x}^N = 0 \qquad (2.27)$$

This is the geodesic equation in affine parameterisation.

Euler-Lagrange equations that generate geodesics:

The Euler-Lagrange equation for the FSM Lagrange function

$$\mathcal{L} = \frac{1}{2}\,g_{MN}\,\dot{x}^M\,\dot{x}^N$$

Is with $g_{MN} = [1 + \cos(k_{Uni}\,t)]\,(\eta_{MN} + h_{MN})$::

$$\ddot{x}^\lambda + \Gamma^\lambda_{MN}\,\dot{x}^M\,\dot{x}^N = 0$$

and is equivalent to the geodesic equation:

$$\frac{d^2x^\lambda}{d^2\lambda^2} + \Gamma^\lambda_{MN} \frac{dx^M}{d\lambda} \frac{dx^N}{d\lambda} = 0 \tag{2.28}$$

- $\frac{d^2x^\lambda}{d^2\lambda^2}$ – acceleration, measures track curvature
- Γ^λ_{MN} – describes a geometric expression of how space-time itself directs the movement of objects; unity: $\frac{1}{m}$
- $\frac{dX^M}{d\lambda}$ bzw. $\frac{dX^N}{d\lambda}$ – tangential vector
- $\frac{dX^4}{d\tau}$ corresponds specifically to V_4 from **Kapitel 1.2**
- $\frac{dX^5}{d\tau}$ corresponds specifically to V_5 from **Kapitel 1.2**

In FSM, the time- and cosine-dependent terms in g_{MN} make the Christoffel symbols dynamic. This is the key to unifying the four fundamental forces and leads to new effects such as coupling frequencies.

Reduction to 4D and FSM-specific effects:

In the limiting cases (compact → 0) the compact wave-field dimensions (dy_4, dy_5, dy_6) are integrated and averaged. Following reduction to GTR, effective fields and scalars ϕ are obtained:

$$\frac{d^2X^\mu}{d^2\tau^2} + \Gamma^\mu_{\nu p} \frac{dX^\nu}{d\tau} \frac{dX^\rho}{d\tau} = 0$$ with: Γ from $g^{eff}_{\mu\nu}$ inclusive half r_s due to $\cos(kt)$

- $g^{eff}_{\mu\nu}$ – effective 4-dimensional metric provides observable physics
- ν, ρ – Counting variables for 4D space-time directions, they 'count' the possible velocity combinations. They are submitted to $\Gamma^\mu_{\nu\rho}$.

The additional terms of the FSM extension generate the quantum effects.

3. Christoffel symbols in FSM

Christoffel symbols are the central affine connection in space-time and describe how vectors change under parallel transport. In the FSM, they arise from the oscillating metric and couple global cosmic modulations, local gravitational disturbances and vector fields. They are time-dependent and form the basis for the geodesic equation. The metric is given by

$ds^2 = g_{MN}\, dx^M\, dx^N$

$ds^2 = [1 + \cos(k_{Uni}\, t)]\, [- c^2\, dt^2 + dx^2 + dy^2 + dz^2 + dy_4^2 + dy_5^2 + dy_6^2 + \frac{G\, M}{c^2\, r}\, (1 + \cos(kt + \beta))\, (c^2\, dt^2 + dx^2 + dy^2 + dz^2) + 2(A_\mu^a + \delta A_\mu^a)\, dy_a\, dx^\mu]$

General formula for Christoffel symbols Γ of the second kind and perturbation approach:

$$\Gamma^\lambda_{MN} = \frac{1}{2}\, g^{\lambda P}\, (\partial_M g_{NP} + \partial_N g_{MP} - \partial_P g_{MN}) \qquad (2.29)$$

- Γ^λ_{MN} – describes the change in vectors during parallel transport; unit: $\frac{1}{\text{m}}$
- $\partial_M = \frac{\partial}{\partial x^M}$ – is the derivative; unit: $\frac{1}{\text{m}}$
- $g^{\lambda P}$ – inverse metric; dimensionless
- Indices: λ, M, N, P = 0, 1, 2, 3, 4, 5, 6

The expression follows from the requirement that the covariant derivative of the metric tensor vanishes ($\nabla_P\, g_{MN} = 0$).

The metric is defined as

$g_{MN} = [1 + \cos(k_{Uni}\, t)]\, (\eta_{MN} + h_{MN})$

The disturbance involving h_{MN} is treated perturbatively. It is approximated by $\Gamma \approx \Gamma_0 + \delta\Gamma$, whereas Γ_0 is assumed to be flat:

$$\Gamma^\lambda_{MN} \approx \frac{1}{2}\, \eta^{\lambda P}\, (\partial_M\, h_{NP} + \partial_N\, h_{MP} - \partial_P\, h_{MN}) + \text{other terms}$$

- Γ^λ_{MN} – addition to the above: codified curvature; λ – guidance; M, N – initial direction
- $\frac{1}{2}$ – normalisation factor, results in symmetry of Γ
- $\eta^{\lambda P}$ – inverse Minkowski metric with (-1, 1, 1, 1, 1, 1, 1)
- $\partial_M\, h_{NP}$ – derivative; unit: $\frac{1}{\text{m}}$; measures the change in metric in the M-direction
- $\partial_N\, h_{MP}$ – symmetric term, records changes in the metric in the N-direction
- $-\ \partial_P\, h_{MN}$ – subtraction corrects for the P-direction and ensures freedom from torsion

Inversion of the metric for further calculation:

From the diagonal metric (point 1), in a linear approximation, because $h_{MN} << 1$ is small:

$$g^{MN} \approx \frac{1}{1+\cos(k_{Uni}\, t)} (\eta^{MN} - h^{MN}) \tag{2.30}$$

Whereas:

$h^{MN} = \eta^{MP}\, \eta^{QN}\, h_{PQ}$

- η^{MN} – inverse of the full 7-dimensional Minkowski metric; dimensionless
- g^{MN} – inverse metric, perturbative; dimensionless
- Indices: M, N, P, Q = 0, 1, 2, 3, 4, 5, 6

Elements of the Christoffel symbols in the FSM:

$$\Gamma^{\lambda}_{MN} = \Gamma^{\lambda\,(\text{global})}_{MN} + \Gamma^{\lambda\,(\text{local})}_{MN} + \Gamma^{\lambda\,(\text{Vector})}_{MN} \tag{2.31}$$

- Global: cosmic oscillation with $[1 + \cos(k_{Uni}\, t)]$
- Local: local oscillation; GTR-like: classical gravitational curvature with: $(1 + \cos(kt + \beta))$
- Vector: Coupling via vector fields with $2(A^a_\mu + \delta A^a_\mu)\, dy_a\, dx^\mu$

Calculation of Christoffel symbols for the global component $\Gamma^{\lambda\,(\text{global})}_{MN}$:

$$\Gamma^{\lambda\,(\text{global})}_{MN} = \frac{1}{2}\, \eta^{\lambda P}\, (\partial_M [\cos(k_{Uni}\, t)]\, \eta_{NP} + \partial_N [\cos(k_{Uni}\, t)]\, \eta_{MP} - \partial_P [\cos(k_{Uni}\, t)]\, \eta_{MN})$$

Given that the derivative ∂_M is non-zero only for the time coordinate $M = 0$ and

$\partial_0 [\cos(k_{Uni}\, t)] = - k_{Uni} \sin(k_{Uni}\, t)$,

the expression simplifies to:

$$\boldsymbol{\Gamma^{\lambda\,(\text{global})}_{MN} = -\frac{1}{2}\, k_{Uni} \sin(k_{Uni}\, t)\, (\eta^{\lambda}_{N}\, \delta^{0}_{M} + \eta^{\lambda}_{M}\, \delta^{0}_{N} - \eta_{MN}\, \eta^{\lambda 0})} \tag{2.32}$$

- $\eta^{\lambda}_{N}\, \delta^{0}_{M}$ or $\eta^{\lambda}_{M}\, \delta^{0}_{N}$ – terms generate a time-dependent acceleration in the time direction (global ‘acceleration’ and ‘deceleration’ of the entire space-time)
- $- \eta_{MN}\, \eta^{\lambda 0}$ – minusterm corrects the spatial components and ensures consistency with the metric inversion
- $\eta^{\lambda P}$ and η_{MN} – are the scalar Minkowski components; dimensionless
- k_{Uni} – cosmic angular frequency $k_{Uni} = \frac{c}{r_{Uni}}$
- Indices: M, N, P, λ = 0, 1, 2, 3, 4, 5, 6

This expression is substituted directly into the geodesic equation in the FSM, generating the global pulsating force that modulates the entire universe.

Calculation of Christoffel symbols for the local component $\Gamma^{\lambda\,(\text{local})}_{MN}$:

The local term arises solely from the derivative of the local gravitational perturbation with:

$h_{\mu\nu} = \frac{GM}{c^2\,r}\,(1 + \cos(kt + \beta))\,\delta_{\mu\nu}$ with: $h_{mn} = 0$; $h_{\mu m} = 0$ with a purely local component

Perturbative 7D:

$$\boldsymbol{\Gamma^{\lambda}_{MN} \approx \frac{1}{2}\,\eta^{\lambda P}\,(\partial_M h_{NP} + \partial_N h_{MP} - \partial_P h_{MN})}$$

Given that $h_{\mu\nu}$ is non-zero only for μ, ν = 0, 1, 2, 3 (visible 4D block), the expression reduces to the 4D components with:

$$\Gamma^{\sigma}_{\mu\nu} \approx \frac{1}{2}\,\eta^{\sigma\rho}\,(\partial_\mu h_{\nu\rho} + \partial_\nu h_{\mu\rho} - \partial_\rho h_{\mu\nu}) \qquad (2.33)$$

- $\eta^{\sigma\rho}$ – inverse Minkowski with diag(-1, 1, 1, 1); for a flat basis
- $h_{\mu\nu}$ – dynamic term, generates gravity with: $h_{\mu\nu} = \frac{GM}{c^2\,r}\,\delta_{\mu\nu}$

For the majority share Γ^{r}_{tt}, radial, weak field, $r \gg \frac{GM}{c^2}$ with $-\,\partial_\rho h_{\mu\nu}$:

$$\Gamma^{r}_{tt} = -\frac{1}{2}\,\partial_r\left[\frac{GM}{c^2\,r}\,(1 + \cos(kt + \beta))\right]$$

$$\Gamma^{r}_{tt} = \frac{1}{2}\,\frac{GM}{c^2\,r^2}\,(1 + \cos(kt + \beta)) \qquad (2.34)$$

- ∂_r – radial discharge according to $(\frac{1}{r})$ results in a field strength ~ $(\frac{1}{r^2})$
- $\Gamma^{r}_{tt} = \frac{1}{2}\,\frac{GM}{c^2\,r^2}$ – GTR solution; produces purely static curvature
- $(1 + \cos(kt + \beta))$ – the mean reproduces Newtonian gravity, but dynamically within the FSM; see the approach in **Chapter 2.3**, ‘Sine Periodicity’
- $\partial_r h_{tt}$ – provides the radial gradient of the local oscillation and generates the attractive curvature, which is modulated by the factor $(1 + \cos(kt + \beta))$
- Other derivatives, e.g. ∂t terms, give rise to additional time-dependent contributions which describe the oscillating force in the geodesic equation.

Calculation of Christoffel symbols for the vector component $\Gamma^{\lambda\,(\text{Vector})}_{MN}$:

The vector coupling term arises from the mixed metric term of the FSM:

$2(A^a_P + \delta A^a_P)\, dy_a\, dx^\mu$

$\Gamma^\lambda_{MN} = \frac{1}{2} g^{\lambda P} (\partial_M g_{NP} + \partial_N g_{MP} - \partial_P g_{MN})$ with: $g^{\lambda P} \approx \eta^{\lambda P}$ in a linear approximation

$$\Gamma^\lambda_{MN} \approx \frac{1}{2} g^{\lambda P}\, \partial_M [2(A^a_P + \delta A^a_P)\, dy_a\, dx^\mu]\, \delta_{aN} \quad (2.35)$$

- $g^{\lambda P} \approx \eta^{\lambda P}$; flat inverse metric in linear order
- ∂_M – derivative with respect to a coordinate M; causes the field strength $F_{MP} = \partial_M A_P - \partial_P A_M$
- A^a_P – background vector field
- δA^a_P – small fluctuations/disturbances in the vector field
- δ_{aN} – Kronecker delta; projected onto the direction of the compact wave-field dimensions or the vector index a

Explicitly 7D for all components:

$$\boldsymbol{\Gamma^{\lambda\,(\text{Vector})}_{MN} = \frac{1}{2} g^{\lambda P} [\partial_M (A^a_P + \delta A^a_P) + \partial_N (A^a_M + \delta A^a_M) - \partial_P (A^a_M + \delta A^a_N)]} \quad (2.36)$$

Explicitly 4D for the interaction/coupling between 4D-visible and 3D-compact:

$$\Gamma^{\lambda\,(\text{Vector})}_{\mu a} = \frac{1}{2} g^{\lambda P} [\partial_\mu (A^a_P + \delta A^a_P) + \partial_a (A^a_\mu + \delta A^a_\mu) - \partial_P (A^a_\mu + \delta A^a_\mu)]$$

Predominant part:

$$\Gamma^{\lambda\,(\text{Vector})}_{\mu a} = \frac{1}{2} \eta^{\lambda P}\, \partial_\mu (A^a_P + \delta A^a_P)$$

- This term describes the interaction of the vector fields between the visible 4D space-time (0, 1, 2, 3) and the compact wave-field dimensions (4, 5, 6).
- δA^a_P – disturbance term; provides local counteracting forces during deformation
- Indices: λ, P = 0, 1, 2, 3, 4, 5, 6; μ, ν = 0, 1, 2, 3; a = 4, 5, 6

Results for the Christoffel symbols in the FSM:

The Christoffel symbols in the FSM are time-dependent and contain both global and local oscillations.

$$\Gamma^\lambda_{MN} = \Gamma^{\lambda\,(\text{global})}_{MN} + \Gamma^{\lambda\,(\text{local})}_{MN} + \Gamma^{\lambda\,(\text{Vector})}_{MN}$$

- $\Gamma^{\lambda\,(\text{global})}_{MN} = -\frac{1}{2}\, k_{Uni} \sin(k_{Uni}\, t)\, (\eta^{\lambda}_{N}\, \delta^{0}_{M} + \eta^{\lambda}_{M}\, \delta^{0}_{N} - \eta_{MN}\, \eta^{\lambda 0})$
- $\Gamma^{\lambda\,(\text{local})}_{MN} \approx \frac{1}{2}\, \eta^{\lambda P}\, (\partial_M h_{NP} + \partial_N h_{MP} - \partial_P h_{MN})$
- $\Gamma^{\lambda\,(\text{Vector})}_{MN} = \frac{1}{2}\, g^{\lambda P}\, [\partial_M (A^{a}_{P} + \delta A^{a}_{P}) + \partial_N (A^{a}_{M} + \delta A^{a}_{M}) - \partial_P (A^{a}_{M} + \delta A^{a}_{N})]$

The main components for 4D: (2.37)

$$\Gamma^{\lambda}_{MN} = \frac{1}{2}\frac{GM}{c^2 r^2}\,[1 + \cos(kt + \beta)] + \frac{1}{2}\,\eta^{\lambda P}\,\partial_\mu\,(A^{a}_{P} + \delta A^{a}_{P}) - \frac{1}{2}\,k_{Uni}\sin(k_{Uni}\,t)\,(\eta^{\lambda}_{N}\,\delta^{0}_{M} + \eta^{\lambda}_{M}\,\delta^{0}_{N} - \eta_{MN}\,\eta^{\lambda 0})$$

4. The Riemann tensor in FSM

The Riemann tensor measures the intrinsic curvature of space-time by quantifying vectors that change when transported in parallel around a closed loop. It is a measure of whether a space is curved or flat. In flat spaces such as Minkowski space, the Riemann tensor disappears completely. For FSM, the Riemann tensor is extended to capture curvature not only in 4-dimensional observable space-time, but also in compact dimensions in the wave-field. This is essential for the field equations. In the FSM, it includes time-dependent contributions from global cosmic oscillations, local gravitational perturbations and vector coupling.

Parallel transport of a vector V^{σ}:

A vector V^{P} is transported parallel to a curve with tangent vector $u^{M} = \frac{dx^{M}}{d\lambda}$ if its covariant derivative vanishes:

$$\nabla_M V^{\rho} = \partial_M V^{\rho} + \Gamma^{P}_{MQ}\, V^{Q} = 0$$

If the same vector is transported along two different paths around an infinitesimally small loop (an area with sides of length δx^{M} and δx^{N}), a difference ΔV^{ρ} results. This difference defines the Riemann tensor:

$$\Delta V^{\rho} = -\, R^{P}_{QMN} V^{Q}\, \delta x^{M}\, \delta x^{N} \quad (2.38)$$

- V^{ρ} – displacement vector; unit: m
- R^{P}_{QMN} – Riemann tensor; unit: $\frac{1}{m^2}$; quantifies curvature; indices: P up is the direction of the vector; Q is the vector index below; M, N are the grinding directions
- Indices: P, Q above – contravariant; P, Q = 0, …, 6; vector components
- Indices: M, N = 0, …, 6; directions of the loop

Definition – Riemann tensor from Christoffel symbols 7D:

$$R^P_{QMN} = \partial_M \Gamma^P_{NQ} - \partial_N \Gamma^P_{MQ} + \Gamma^P_{ML}\, \Gamma^L_{NQ} - \Gamma^P_{NL}\, \Gamma^L_{MQ} \quad (2.39)$$

- ∂_M – partial derivative with respect to x^M; unit: $\frac{1}{m}$; measures the change in the Γ; index *M* below corresponds to the direction
- Γ^P_{NQ} – Christoffel symbol; unit: $\frac{1}{m}$; joining term; indices: *P* on top is the goal; *N*, *Q* below correspond to the source
- $\Gamma^P_{ML}\, \Gamma^L_{NQ}$ – a square correction arising from the non-commutative nature of the connection; unity: $\frac{1}{m^2}$; introduces non-linear terms that generate curvature via Γ-interactions; *L* is summed over the index 0,…, 6
- $- \Gamma^P_{NL}\, \Gamma^L_{MQ}$ – a square correction arising from the non-commutative nature of the connection; induces antisymmetry in *M, N* for curvature
- Indices of R^P_{QMN} – *P* vector is shown above; *Q* below is the vector index; *M*, *N* the subscripts indicate the directions; antisymmetric

Perturbative approximation in FSM:

In a linear approximation where $h_{MN} << 1$ and using the Christoffel symbols already derived, the Riemann tensor simplifies. In the first order with linear curvature, ΓΓ is neglected for weak fields:

$$R^P_{QMN} \approx \partial_M \Gamma^P_{NQ} - \partial_N \Gamma^P_{MQ} \quad (2.40)$$

The Christoffel symbols consist of:

$$\Gamma^\lambda_{MN} = = \Gamma^{\lambda\,(\text{global})}_{MN} + \Gamma^{\lambda\,(\text{local})}_{MN} + \Gamma^{\lambda\,(\text{Vector})}_{MN}$$

Specific contributions from Γ^λ_{MN}:

Example: a radial term in a weak field, to be detected in the visible universe

Global contribution (cosmic oscillation):

$$R^{P\,(\text{global})}_{QMN} \approx \partial_M \left(- \frac{1}{2}\, k_{Uni} \sin(k_{Uni}\, t)\, \eta^P_Q\, \delta^N_0\right) - \partial_N \left(- \frac{1}{2}\, k_{Uni} \sin(k_{Uni}\, t)\, \eta^P_Q\, \delta^M_0\right) \quad (2.41)$$

Local contribution (gravitational disturbance):

$$R^{r\,(\text{local})}_{QMN} \approx \partial_r \left(\frac{1}{2} \frac{GM}{c^2\, r^2} (1 + \cos(kt + \beta))\right) = - \frac{2}{2} \frac{GM}{c^2\, r^3} (1 + \cos(kt + \beta))$$

$$R_{QMN}^{r\,(\text{local})} = -\frac{GM}{c^2\, r^3}\,(1 + \cos(kt + \beta)) \tag{2.42}$$

- $\partial_r h_{tt}$ → contributes to Γ_{tt}^{r}
- $\partial_r h_{rr} \approx 0$
- $\partial_r h_{tr} \approx 0$
- $\partial_r h_{rt} \approx 0$

Vector contribution (coupling):

$$R_{\mu ab}^{\lambda\,(\text{Vector})} \approx \partial_\mu \left(\frac{1}{2}\,\eta^{\lambda P}\,\partial_a\,(A_P^{a} + \delta A_P^{a})\right) \tag{2.43}$$

- $\frac{G\,M}{c^2\,r^3}$ – spherically symmetric term; results in higher moments of curvature
- $R_{\mu ab}^{\lambda\,(\text{Vector})}$ – describes in detail the interaction between visible 4D space and the compact 3D wave-field dimensions
- Indices: λ – event component, $\lambda = 0, \ldots, 6$; μ below is a visible 4D index, $\mu = 0, 1, 2, 3$; a, b – first and second compact indices, $a, b = 4, 5, 6$; ρ – summary index, $\rho = 0, \ldots, 6$; a in A_P^{a} – indicates, which of the three vector fields A^4, A^5, A^6 originates from the compact dimensions

Reduction from 7D to 4D:

After integration over the compact dimensions y_4, y_5, y_6 and averaging, the effective 4D Riemann tensor is obtained:

$$R_{\sigma\mu\nu}^{\rho} = R_{\sigma\mu\nu}^{\rho\,(\text{global})} + R_{\sigma\mu\nu}^{\rho\,(\text{local})} + R_{\sigma\mu\nu}^{\rho\,(\text{Vector})}$$

5. The Ricci tensor in FSM

The Ricci tensor describes a contraction of the Riemann tensor from point 4 and reduces the local curvature from a 4-tensor form to a symmetric 2-tensor form. This tensor measures the average curvature in every direction and is crucial to the Einstein field equations, which relate the energy density and momentum of matter. In FSM, the Ricci tensor is time-dependent.

Contraction of the Riemann tensor:

The Ricci tensor is formed by contracting the first and third indices of the Riemann tensor, i.e. summing over a common index. In full form:

$$R_{MN} = \sum_P R_{MPN}^{P} \tag{2.44}$$

- R_{MN} – Ricci tensor, unit: $\frac{1}{m^2}$; causes combined curvature in directions *M* and *N*
- R^P_{MPN} – Riemann tensor; unit: $\frac{1}{m^2}$; causes complete curvature; indices here: *P* top/bottom is contracted, summed over $P = 0,\ldots, 6$
- $\sum_P$ – summation over *P*; reduced 4-Index-Riemann-tensor to a 2-index tensor that measures the local energy density

Application of Riemann's formula:

$$R^P_{QMN} = \partial_M \Gamma^P_{NQ} - \partial_N \Gamma^P_{MQ} + \Gamma^P_{ML} \Gamma^L_{NQ} - \Gamma^P_{NL} \Gamma^L_{MQ}$$

Contraction over *P* (first and third index):

$$R_{MN} = \partial_P \Gamma^P_{NM} - \partial_N \Gamma^P_{PM} + \Gamma^P_{PL} \Gamma^L_{NM} - \Gamma^P_{NL} \Gamma^L_{PM} \tag{2.45}$$

- $\partial_P \Gamma^P_{NM}$ – derivative and sum, unit: $\frac{1}{m^2}$; measures the change in the connection in *P*-direction; *P* summed up $0,\ldots, 6$
- $- \partial_N \Gamma^P_{PM}$ – subtraction; causes antisymmetry; correction for the *N*-direction
- $\Gamma^P_{PL} \Gamma^L_{NM}$ – product; unit: $\frac{1}{m^2}$; causes non-linear interactions between Γ; *P* summed up
- $- \Gamma^P_{NL} \Gamma^L_{PM}$ – subtraction; causes antisymmetry in the nonlinear terms
- *L* – cumulative running index $\lambda = 0,\ldots, 6$; takes all directions into account in the products

Formula (2.45) is covariant and dynamic in the FSM due to the sine/cosine terms in Γ.

Perturbative approximation in the FSM:

Due to $\Gamma \sim \delta g$ (first order), the second-order Γ products $\sim (\delta g)^2$ are negligible in linear approximation for $h_{MN} << 1$:

$$R_{MN} \approx \partial_P \Gamma^P_{NM} - \partial_N \Gamma^P_{PM} \tag{2.46}$$

As the Christoffel symbols consist of three parts (global, local, vector), the Ricci tensor is given by:

$$R_{MN} = R^{(\text{global})}_{MN} + R^{(\text{local})}_{MN} + R^{(\text{Vector})}_{MN}$$

Each Christoffel component contributes its own Ricci component. The global oscillation modulates the overall curvature of space-time, the local component modulates the gravitational curvature around the mass, and the vector component modulates the coupling to the wave-field.

Contributions and explicit forms:

Global contribution:

$$R_{MN}^{(\text{global})} \approx \partial_P \left(- \frac{1}{2}\, k_{Uni} \sin(k_{Uni}\, t)\, \eta_N^P\, \delta_M^0\right) - \partial_N \left(- \frac{1}{2}\, k_{Uni} \sin(k_{Uni}\, t)\, \eta_P^P\, \delta_M^0\right) \tag{2.47}$$

Local contribution:

$$R_{MN}^{(\text{local})} = \partial_P \left(\frac{1}{2} \frac{GM}{c^2\, r^2} (1 + \cos(kt + \beta))\, \delta_N^r\, \delta_M^t\right) - \partial_N \Gamma_{PM}^P$$

$$R_{MN}^{(\text{local})} \approx \partial_P \left(\frac{1}{2} \frac{GM}{c^2\, r^2} (1 + \cos(kt + \beta))\, \delta_N^r\, \delta_M^t\right) \tag{2.48}$$

- $\partial_N \Gamma_{PM}^P \approx 0$ for low-field, radial, $M = N = t$

Vector contribution:

$$R_{MN}^{(\text{Vector})} = \partial_P \left(\frac{1}{2}\, \eta^{P\rho}\, \partial_M \left(A_\rho^a + \delta A_\rho^a\right) \delta_a^N\right) \tag{2.49}$$

- δ_a^N – Kronecker delta; isolates the components of the Riemann tensor with $N \in \{4, 5, 6$ that relate to the compact wave-field dimensions, and prevents this vectorial contribution from occurring directly in pure 4D (μ, ν = 0, 1, 2, 3). This shifts the cause of the interaction/coupling into the wave-field.
- Sum index: ρ = 0,…, 6; contracts the inverse metric using the inner derivative

Reduced to an effective 4D:

By reducing the indices M and N to μ and ν = 0, 1, 2, 3, the 4D solution of Ricci is obtained:

$$R_{\mu\nu}^{(4)} = R_{\mu\nu}^{(\text{global})} + R_{\mu\nu}^{(\text{local})} + R_{\mu\nu}^{(\text{Vector})}$$

Ricci scalar R:

The Ricci scalar is the full contraction of the Riemann tensor with the inverse metric, and gives the local mean curvature of space-time as a scalar value.

$$R = g^{MN}\, R_{MN} = g^{MN}\, R_{MPN}^P \tag{2.50}$$

with: $g^{MN} \approx \dfrac{1}{1+\cos(k_{Uni}\, t)} \left(\eta^{MN} - h^{MN}\right)$

- R – curvature scalar, unit: $\frac{1}{\text{m}^2}$; indicates local scalar curvature; positive for expanding systems, negative for bound systems; dynamic in FSM

- g^{MN} – inverse metric; increases the indices of the Ricci tensor; the indices *M* and *N* above are covariant; *M*, *N* = 0,…, 6; in 7D
- R_{MN} – Ricci tensor; unit: $\frac{1}{m^2}$; summarises curvature; indices *M*, *N* in the bottom are covariant
- Contraction with the implicit summation over *M* and *N* reduces the 2-index Ricci tensor to a scalar that measures the trace of the curvature

Perturbative approximation with $h_{MN} << 1$:

$R \approx \eta^{MN} R_{MN}$

$R = R^{(\text{global})} + R^{(\text{local})} + R^{(\text{Vector})}$

Global contribution:

$$R^{(\text{global})} \approx \eta^{MN} [\partial_P (- \frac{1}{2} k_{Uni} \sin(k_{Uni}\, t)\, \eta^P_N\, \delta^0_M) - \partial_N (- \frac{1}{2} k_{Uni} \sin(k_{Uni}\, t)\, \eta^P_P\, \delta^0_M)]$$

Local contribution:

$$R^{(\text{local})} \approx \eta^{MN} [\partial_P (\frac{1}{2} \frac{GM}{c^2\, r^2} (1 + \cos(kt + \beta))\, \delta^r_N\, \delta^t_M)] = - \frac{GM}{c^2\, r^3} (1 + \cos(kt + \beta))$$

Vector contribution:

$$R^{(\text{Vector})} \approx \eta^{MN} [\partial_P (\frac{1}{2} \eta^{P\rho}\, \partial_M (A^a_\rho + \delta A^a_\rho)\, \delta^N_a)]$$

Reduction to 4D:

The compact wave-field dimensions y_4, y_5, y_6 are integrated and averaged.

$R^{(4)} = R^{(\text{global})} + R^{(\text{local})} + R^{(\text{Vector})}$

6. The Einstein tensor in FSM

The Einstein tensor links the curvature of geometry with the energy-momentum distribution and forms the basis of Einstein's field equations. This tensor combines the Ricci tensor (point 5.) and the scalar curvature term to create a divergence-free form that describes gravity as geometric effects. In FSM, this tensor explains how curvature leads to gravity and other forces.

<u>Definition of the Einstein tensor from the Ricci tensor and scalar:</u>

The Einstein tensor is a linear combination of the Ricci tensor, metric and scalar.

$$G_{MN} = R_{MN} - \frac{1}{2} g_{MN} R \qquad (2.51)$$

- G_{MN} – Einstein tensor; unit: $\frac{1}{m^2}$; links curvature with matter/energy; indices M, N below are covariant; M, N = 0,…, 6; in 7D (0 = time, 1, 2, 3 = visible, 4, 5, 6 = compact)
- R_{MN} – Ricci tensor; unit: $\frac{1}{m^2}$; causes local curvature; indices: M, N
- $\frac{1}{2}$ – normalisation factor; makes Einstein tensor G trace-free
- g_{MN} – metric tensor from point 1.; multiplied scalar to tensor form
- R – curvature scalar for the average curvature; unit: $\frac{1}{m^2}$

The purpose of this definition is to ensure that there is no divergence, thereby guaranteeing the conservation of energy:

$$\nabla^M G_{MN} = 0 \tag{2.52}$$

Inserting the Ricci tensor and scalar from point 5.:

$$G_{MN} = (R_{MN}^{(global)} + R_{MN}^{(local)} + R_{MN}^{(Vector)}) - \frac{1}{2} g_{MN} (R^{(global)} + R^{(local)} + R^{(Vector)}) \tag{2.53}$$

- $- \frac{1}{2} g_{MN}$ () – combination; makes G divergent free by distributing the scalar curvature isotropically in all directions

The tensor is covariant and time-dependent in FSM through sin(kt) or cos(kt) in the Γ.

Perturbative approximation in FSM:

In the linear approximation, ΓΓ ~ $(\delta g)^2$, $g \approx g_0$ is neglected and the following applies $h_{MN} << 1$:

$$G_{MN} \approx R_{MN} - \frac{1}{2} \eta_{MN} R \tag{2.54}$$

Specific contributions:

$$G_{MN} = G_{MN}^{(global)} + G_{MN}^{(local)} + G_{MN}^{(Vector)}$$

Global contribution:

$$G_{MN}^{(global)} = R_{MN}^{(global)} - \frac{1}{2} \eta_{MN} R^{(global)} \tag{2.55}$$

- $R_{MN}^{(global)} \approx \partial_P (- \frac{1}{2} k_{Uni} \sin(k_{Uni}\, t)\, \eta_N^P\, \delta_M^0) - \partial_N (- \frac{1}{2} k_{Uni} \sin(k_{Uni}\, t)\, \eta_P^P\, \delta_M^0)$
- $R^{(global)} \approx \eta^{MN} [\partial_P (- \frac{1}{2} k_{Uni} \sin(k_{Uni}\, t)\, \eta_N^P\, \delta_M^0) - \partial_N (- \frac{1}{2} k_{Uni} \sin(k_{Uni}\, t)\, \eta_P^P\, \delta_M^0)]$

Local contribution:

$$G_{MN}^{(\text{local})} = R_{MN}^{(\text{local})} - \frac{1}{2}\,\eta_{MN}\,R^{(\text{local})} \tag{2.56}$$

- $R_{MN}^{(\text{local})} = \partial_P\,(\frac{1}{2}\,\frac{GM}{c^2\,r^2}\,(1 + \cos(kt + \beta))\,\delta_N^r\,\delta_M^t) - \partial_N \Gamma_{PM}^P$
- $\partial_N \Gamma_{PM}^P \approx 0$ for low-field, radial, $M = N = t$
- $R^{(\text{local})} \approx \eta^{MN}\,[\partial_P\,(\frac{1}{2}\,\frac{GM}{c^2\,r^2}\,(1 + \cos(kt + \beta))\,\delta_N^r\,\delta_M^t)]$

Vector contribution:

$$G_{MN}^{(\text{Vector})} = R_{MN}^{(\text{Vector})} - \frac{1}{2}\,\eta_{MN}\,R^{(\text{Vector})} \tag{2.57}$$

- $R_{MN}^{(\text{Vector})} = \partial_P\,(\frac{1}{2}\,\eta^{P\rho}\,\partial_M\,(A_\rho^a + \delta A_\rho^a)\,\delta_a^N)$
- $R^{(\text{Vector})} \approx \eta^{MN}\,[\partial_P\,(\frac{1}{2}\,\eta^{P\rho}\,\partial_M\,(A_\rho^a + \delta A_\rho^a)\,\delta_a^N)]$

The absence of divergence, as shown by the Bianchi identities, ensures the conservation of energy:

The second Bianchi identities are geometric identities for the Riemann tensor in torsion-free spaces and are given by:

$$\nabla_R\,R_{QMN}^P + \nabla_M\,R_{QNR}^P + \nabla_N\,R_{QRM}^P = 0 \tag{2.58}$$

- ∇_R – covariate derivative with respect to R; unit: $\frac{1}{\text{m}}$; takes curvature into account; index R below is covariant; $R = 0,\ldots, 6$ in 7D
- R_{QMN}^P – Riemann tensor; unit: $\frac{1}{\text{m}^2}$; full curvature, indices: P above is Vector; Q below is the vector index; M, N directions are given below
- $\nabla_R\,R_{QMN}^P$ or $\nabla_M\,R_{QNR}^P$ or $\nabla_N\,R_{QRM}^P$ – cyclic permutation results in antisymmetry in R, M, N
- $\nabla_N\,R_{QRM}^P$ – closes the cycle; implies identity with zero, i.e. the curvature is consistent
- 0 – zero tensor; due to geometric necessity
- Indices: P upper free index, Q lower free index, M, N, R are loop directions; P, Q, M, N, $R = 0,\ldots, 6$

These identities always hold for any affine transformation (including in the FSM with oscillations and vector fields). They are a purely geometric property of the Christoffel symbols and state that the curvature is 'cyclic'.

Contraction across the first and third index ($P = Q$):

$$\nabla_R R^P_{MPN} + \nabla_M R^P_{NPR} + \nabla_N R^P_{RPM} = 0$$

The contracted form is:

$$\nabla_R R^R_{MN} + \nabla_M R - \nabla_N R^R_{MR} = 0 \qquad \text{with: } R = R^P_P \text{ as the Ricci scalar}$$

Substituting into the Einstein tensor:

$$G_{MN} = R_{MN} - \frac{1}{2} g_{MN} R$$

The contracted Bianchi identity is multiplied by g^{MN}, and the metric compatibility condition $\nabla_g = 0$ is applied:

$$\nabla^M G_{MN} = \nabla^M R_{MN} - \frac{1}{2} \nabla_N R - \frac{1}{2} g_{MN} \nabla^M R + \frac{1}{2} \nabla_N R = 0$$

$$\nabla^M G_{MN} = 0 \qquad (2.59)$$

- $\nabla^M G_{MN}$ – covariate divergence; unit: $\frac{1}{m^3}$; results in the maintenance rate

The absence of divergence ensures that the left-hand side of the field equation

$$G_{MN} = \frac{8\pi G_7}{c^4} T_{MN}$$

automatically enforces local momentum-energy conservation in the FSM for:

$$\nabla^M T_{MN} = 0$$

It is shown that the Einstein tensor G is divergence-free in FSM geometry and that conservation is guaranteed despite rotations.

Reduction for 4D with $\mu, \nu = 0, 1, 2, 3$:

After integration over the compact dimensions y_4, y_5, y_6 and averaging, the effective 4D reduction is obtained:

$$\nabla^\mu G^{(4)}_{\mu\nu} = 0$$

The oscillating terms and vector fields do not violate these identities, because the Bianchi identities are geometrically invariant.

7. Impulse energy tensor in the FSM

The momentum-energy tensor in FSM describes the distribution of energy, momentum, and tension in 7-dimensional field-space. It is symmetrical $T_{MN} = T_{NM}$ and divergence-free $\nabla^M T_{MN} = 0$ in order to ensure conservation laws. In FSM, the momentum-energy tensor arises from the electric/gravitational potential of the universe and photon fields. It links geometry with physical effects. This tensor is derived based on the method of the variation principle of GTR and extended to a 7-dimensional model.

Variation principle definition from matter action:

$$T_{MN} = -\frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{MN}} \quad \text{; with: } S_m = \int \mathcal{L}_m \sqrt{-g}\, d^7x \text{ the matter action} \qquad (2.60)$$

- T_{MN} – impulse energy tensor; normalized per volume segment of the wave-field with unit: $\frac{J}{m^3}\frac{1}{m^3} = \frac{J}{m^6} = \frac{kg}{m^4 s^2}$; source of curvature; indices: M, N below are covariant; M, N = 0,…, 6 in 7D (0 = time; 1, 2, 3 = visible; 4, 5, 6 = compact)
- \- 2 – normalization factor; appropriate scaling for field equations
- $\sqrt{-g}$ – root of the metric determinant in 7D; produces an invariant volume element from the metric (point 1.); with $g = \det g_{MN}$
- $\frac{\delta S_m}{\delta g^{MN}}$ – functional normalized variation; unit: $\frac{kg}{m^4 s^2}$; measures the metric dependence of matter; δ corresponds to a small variation
- S_m – action, measure of dynamism; unit: Js; causes the integral over the Lagrangian of matter/fields
- g^{MN} – inverse metric causes variation in the indices above
- d^7x – integral over 7D volume element, including compact dimensions
- $\mathcal{L}_m$ – Lagrange density from fields, in FSM from photon field

The tensor T_{MN} is defined as the response of matter to a change in metric. In FSM, S_m contains geometric fields in the form of sinusoidal oscillations.

Phenomenological form for perfect fluidity, classic basis in FSM:

The perfect fluid is isotropic and viscosity-free:

$$\boldsymbol{T_{MN}^{(classical)} = (\rho + \frac{p}{c^2})\, u_M\, u_N - p\, g_{MN}} \qquad (2.61)$$

This term models the rest energy density (ρ) and pressure (p) along the trajectory (u_M). In FSM, the energy density ρ arises from the electrically internal fion mass within a beam, and p arises from the field tension of the exchange fion in a vacuum. In the FSM, so-called fions (**Chapter 3.2**) take on the physical role of photons in a beam. They are, in comparative terms, similar to an extension of the classical gluon. Furthermore, fions can actively generate a potential relative to the D_{56} dimensional plane, can split by producing an exchange particle and a passive (dark) fion, and can recombine following a field exchange.

- ρ – standardized energy density; unit: $\frac{\text{kg}}{\text{m}^4\text{s}^2}$; total energy per 6D volume (including resting mass)
- p – standardized pressure; unit: $\frac{\text{kg}}{\text{m}^4\text{s}^2}$; is isotropic force; $p = 0$ for dust; $p = \frac{\rho}{3}$ for radiation
- u_M – quadruple pulse; dimensionless without specific index assignment; causes the scalar direction of movement; e.g.: $u^M u_M = - c^2$ with index M below for covariant components (transformed like gradients ∂_M); M above for countervariant component (transformed as differentials dx^M);
- $(\rho + \frac{p}{c^2})$ – enthalpy density
- $(\rho + \frac{p}{c^2})\, u_M\, u_N$ – dynamic term; unit: $\frac{\text{kg}}{\text{m}^4\text{s}^2}$; pulse flow
- $- p\, g_{MN}$ – isotropic pressure in all directions
- g_{MN} – full metric from point 1.; causes projection

Relativistic fields correspond to photons as hollow-body oscillations, with angular momentum generating momentum density. This reproduces classical physics, whereby in FSM the energy density ρ emerges from the geometric mass as resistance to fields.

Extension through compact portion of the wave-field:

Metric:

$$g_{MN} = [1 + \cos(k_{Uni}\, t)]\, (\eta_{MN} + h_{MN})$$

The following applies to the global component with $h_{MN} = 0$:

$$g_{MN}^{\text{global}} = [1 + \cos(k_{Uni}\, t)]\, \eta_{MN}$$

Global Christoffel component:

$$\Gamma^{\lambda\,(\text{global})}_{MN} = - \frac{1}{2}\, k_{Uni} \sin(k_{Uni}\, t)\, (\eta^{\lambda}_{N}\, \delta^{0}_{M} + \eta^{\lambda}_{M}\, \delta^{0}_{N} - \eta_{MN}\, \eta^{\lambda 0})$$

Global Ricci tensor:

$$R^{(\text{global})}_{MN} \approx \partial_P \left(- \frac{1}{2}\, k_{Uni} \sin(k_{Uni}\, t)\, \eta^{P}_{N}\, \delta^{0}_{M}\right) - \partial_N \left(- \frac{1}{2}\, k_{Uni} \sin(k_{Uni}\, t)\, \eta^{P}_{P}\, \delta^{0}_{M}\right)$$

$$R^{(\text{global})}_{MN} = \left(- \frac{1}{2}\, k^{2}_{Uni} \cos(k_{Uni}\, t)\right) (\eta_{MN}\, \eta^{00} - \eta^{0}_{M}\, \eta^{0}_{N} - \eta^{0}_{N}\, \eta^{0}_{M})$$

- $\eta_{MN}\, \eta^{00} = - \eta_{MN}$
- $- \eta^0_M\, \eta^0_N - \eta^0_N\, \eta^0_M$ – time projectors

Global Ricci scalar:

$$R^{\text{global}} = \eta^{MN} R^{(\text{global})}_{MN} = \left(- \frac{1}{2}\, k^2_{Uni} \cos(k_{Uni}\, t)\right) \eta^{MN} \left(\eta_{MN}\, \eta^{00} - \eta^0_M\, \eta^0_N - \eta^0_N\, \eta^0_M\right)$$

- $\eta^{MN}\, \eta_{MN}\, \eta^{00} = - 7$
- $- \eta^{MN}\, \eta^0_M\, \eta^0_N = 1$
- $- \eta^{MN}\, \eta^0_N\, \eta^0_M = 1$

$$R^{\text{global}} = - \frac{1}{2}\, k^2_{Uni} \cos(k_{Uni}\, t)\, [\,-7 + 1 + 1]$$

$$R^{\text{global}} = \frac{5}{2}\, k^2_{Uni} \cos(k_{Uni}\, t)$$

Substitution into the Einstein tensor G_{MN} with $g_{MN} \approx \eta_{MN}$:

$$G_{MN} = R_{MN} - \frac{1}{2}\, g_{MN}\, R$$

$$G^{(\text{global})}_{MN} = \frac{1}{2}\, k^2_{Uni} \cos(k_{Uni}\, t)\, \eta_{MN} + \frac{1}{2}\, k^2_{Uni} \cos(k_{Uni}\, t)\, (- \eta^0_M\, \eta^0_N - \eta^0_N\, \eta^0_M) - \frac{5}{4}\, k^2_{Uni} \cos(k_{Uni}\, t)\, \eta_{MN}$$

Simplification of time-projection terms:

The time projection terms $(- \eta^0_M\, \eta^0_N - \eta^0_N\, \eta^0_M)$ contribute exactly the amount in the 7D contraction that compensates for the remaining factors. The structure of the projector is chosen such that:

$$\frac{1}{2}\, k^2_{Uni} \cos(k_{Uni}\, t)\, (- \eta^0_M\, \eta^0_N - \eta^0_N\, \eta^0_M) - \frac{5}{4}\, k^2_{Uni} \cos(k_{Uni}\, t)\, \eta_{MN} = 0$$

Isotropic approximation of the projector in linear order for $G^{(\text{global})}_{MN}$:

$$G^{(\text{global})}_{MN} = \frac{1}{2}\, k^2_{Uni} \cos(k_{Uni}\, t)\, \eta_{MN}$$

The reduced momentum-energy tensor is defined as the ‘effective source’ of curvature:

$$T^{(\text{global})}_{MN} = \frac{c^4}{8\pi G}\, \frac{1}{2}\, k^2_{Uni} \cos(k_{Uni}\, t)\, \eta_{MN}$$

$$\boldsymbol{T^{(\text{global})}_{MN} = \frac{c^4}{16\pi G}\, k^2_{Uni} \cos(k_{Uni}\, t)\, \eta_{MN}} \qquad (2.62)$$

- G – Gravitational constant normalised to 7D

Expansion using the vector component of the wave-field:

The coupling component $T_{MN}^{(\text{Vector})}$ arises from the dimensional reduction of the 7-dimensional theory of gravity to 4 dimensions. Specifically, 4 dimensions for the interaction/coupling between the 4-dimensional visible and the 3-dimensional compact:

$$\Gamma_{\mu a}^{\lambda\,(\text{Vector})} = \frac{1}{2}\, g^{\lambda P}\, [\partial_\mu (A_P^a + \delta A_P^a) + \partial_a (A_\mu^a + \delta A_\mu^a) - \partial_P (A_\mu^a + \delta A_\mu^a)]$$

Dominant part:

$$\Gamma_{\mu a}^{\lambda\,(\text{Vector})} = \frac{1}{2}\, \eta^{\lambda P}\, \partial_\mu (A_P^a + \delta A_P^a)$$

The starting point is the Lagrangian density of the Yang-Mills field, as is customary, applied to 7D:

$$\mathcal{L}_{YM}^{(7)} = -\frac{1}{4}\, F_{MN}^a\, F^{aMN}\,,$$

whereas the field strength F is defined as follows:

$$F_{\mu\nu}^a = \partial_\mu (A_\nu^a + \delta A_\nu^a) - \partial_\nu (A_\mu^a + \delta A_\mu^a) + g_M^{(7)}\, f^{abc}\, A_M^b\, A_N^c$$

- Indices: M, N = 0,..., 6; calibration group index a = 4,.5, 6 for SU(3), summed over a
- A_ν^a or A_μ^a – 7D calibration vector
- Indices: M, N break down into μ, ν = 0, 1, 2, 3
- $g_M^{(7)}$ – coupling
- f^{abc} – structural constant

$$\mathcal{L}_{YM}^{(4)} = -\frac{1}{4}\,\frac{1}{g_{(YM)}^2}\, F_{\mu\nu}^a\, F^{a\mu\nu} = -\frac{1}{4}\,\frac{1}{\pi}\, F_{\mu\nu}^a\, F^{a\mu\nu}, \tag{2.63}$$

- $g_{(YM)}^2 = g_{eff}^2$ – effective 4D coupling, normalised to π
- $g_{eff}^2 = \dfrac{g_{(7)}^2}{V_{(3)}} = \pi$

$$\mathcal{L}_{YM}^{(4)} = -\frac{1}{4}\,\frac{1}{g_{(YM)}^2}\, F_{\mu\nu}^a\, F^{a\mu\nu} = -\frac{1}{4}\,\frac{1}{\pi}\, F_{\mu\nu}^a\, F^{a\mu\nu}$$

Yang–Mills expression for the reduced case, where the interaction acts as a coupling effect between the 4-dimensional visible and 3-dimensional compact dimensions:

$$T_{\mu\nu}^{(\text{Vector})} = \frac{1}{4\pi} \left(F_{\mu}^{a\lambda}\, F_{a\lambda\rho} - \frac{1}{4}\, g_{\mu a}\, F_{\rho\sigma}^{a}\, F^{a\rho\sigma}\right) \eta^{\rho\nu} \tag{2.64}$$

- $\frac{1}{4\pi}$ – corresponds to a specific normalisation of the calibration coupling
- Indices:
  μ = 0, 1, 2, 3; direction in visible 4D
  ν = 0, 1, 2, 3; free, visible index finger after contraction with $\eta^{\rho\nu}$
  a = 4, 5, 6; compact group index for a vector field A^4, A^5, A^6
  λ, ρ, σ, K = 0,…, 6; sum indices over 7D
- $F_{\mu}^{a\lambda} = \eta^{\lambda\rho}\, F_{\mu\rho}^{a} = \eta^{\lambda\rho}\, [\partial_{\mu}\, (A_{\rho}^{a} + \delta A_{\rho}^{a}) - \partial_{\rho}\, (A_{\mu}^{a} + \delta A_{\mu}^{a})]$
- $F_{a\lambda\rho} = \partial_{\lambda}\, (A_{a\rho} + \delta A_{a\rho}) - \partial_{\rho}\, (A_{a\lambda} + \delta A_{a\lambda})$
- $F_{\rho\sigma}^{a} = \partial_{\rho}\, (A_{\sigma}^{a} + \delta A_{\sigma}^{a}) - \partial_{\sigma}\, (A_{\rho}^{a} + \delta A_{\rho}^{a})$
- $F^{a\rho\sigma} = \eta^{\rho\lambda}\, \eta^{\sigma K}\, F_{\mu K}^{a} = \eta^{\rho\lambda}\, \eta^{\sigma K}\, [\partial_{\lambda}\, (A_{K}^{a} + \delta A_{K}^{a}) - \partial_{K}\, (A_{\lambda}^{a} + \delta A_{\lambda}^{a})]$
- $\eta^{\rho\nu}$ = diag(-1, 1, 1, 1, 1, 1, 1)$^{\rho\nu}$

For the full 7D case:

$$\boldsymbol{T_{MN}^{(\text{Vector})} = \frac{1}{4\pi} \left(F_{M}^{\lambda a}\, F_{Na\lambda} - \frac{1}{4}\, g_{MN}\, F_{\rho\sigma}^{a}\, F^{a\rho\sigma}\right)}$$

<u>Extension to 7D with FSM-specific contributions</u>:

$$T_{MN} = T_{MN}^{(\text{classical})} + T_{MN}^{(\text{global})} + T_{MN}^{(\text{Vector})} \tag{2.65}$$

$$T_{MN} = \left[\left(\rho + \frac{p}{c^2}\right) u_M\, u_N - p\, g_{MN}\right] + \left[\frac{c^4}{16\pi G}\, k_{Uni}^2 \cos(k_{Uni}\, t)\, \eta_{MN}\right] + \left[\frac{1}{4\pi} \left(F_{M}^{\lambda a}\, F_{Na\lambda} - \frac{1}{4}\, g_{MN}\, F_{\rho\sigma}^{a} F^{a\rho\sigma}\right)\right]$$

- T_{MN} – momentum-energy tensor; unit: $\frac{\text{kg}}{\text{m}^4\text{s}^2}$; normalised with 6D volume
- $T_{MN}^{(\text{classical})}$ – classical term; unit: $\frac{\text{kg}}{\text{m}^4\text{s}^2}$; visible matter; indices: M, N = 0,…, 6
- $T_{MN}^{(\text{global})}$ – global term; unit: $\frac{\text{kg}}{\text{m}^4\text{s}^2}$; includes dark energy or invisible photons (**Chapters 2.3** and **3.2**) arising from oscillations; indices: M, N = 0,…, 6; the only relevant metric is the Minkowski metric η_{MN}
- k – Angular frequency, time normalisation from $\frac{1}{\text{s}^2}$ to $\frac{1}{\text{m}^2}$; can be interpreted as the number of turns
- G – Gravitational constant; 4D unit: G_{eff} = 6,674 10^{-11} $\frac{\text{m}^3}{\text{kg s}^2}$; is available in a 7D dilution; G = 6,674 10^{-11} $\frac{\text{m}^6}{\text{kg s}^2}$

- $T_{MN}^{(\text{Vector})}$ – coupling fields from rotations; unit: $\frac{\text{kg}}{\text{m}^4\text{s}^2}$; describes the energy, momentum and potentials of non-Abelian gauge fields (e.g. fermions, bosons); indices: M, N = 0,…, 6
- /4π – standardisation; results in Gaussian units; from: $g_{eff}^2 = \frac{g_{(7)}^2}{V_{(3)}} = \pi$
- $F_M^{\lambda a}\, F_{Na\lambda}$ – non-tracked section; unit: $\frac{\text{kg}}{\text{m}^4\text{s}^2}$; generates energy flow and tension in the tuning forks
  in T_{00}: main contribution to the energy density of the calibration fields
  in T_{0i}: main contribution to the momentum flux of the calibration fields
- $-\frac{1}{4}\, g_{MN}\, F_{\rho_\sigma}^a\, F^{a\rho_\sigma}$ – trace unit of energy density; unit: $\frac{\text{kg}}{\text{m}^4\text{s}^2}$; generates electromagnetic and scalar fields; contributes the isotropic term to the metric g_{MN}, which gives the pressure a negative sign $p = -\rho$ and keeps the tensor trace-free

The derivation shows how the momentum-energy tensor T in the FSM follows directly from the geometric interpretation of mass and energy as field deformation and cosine-shaped rotation (*kt*). Matter is therefore not a fundamental concept, but an emergent effect of resistance to dynamic fields.

8. Calibration potential A_μ^a based on the FSM particle model

The calibration potential A_μ^a is the fundamental mediator of the electromagnetic and weak interactions in the FSM. It arises directly from the off-diagonal components of the 7D metric and mediates between the visible dimensions (μ = 0, 1, 2, 3: time and particle-field $F_{1\text{-}3}$) and the three compact wave-field dimensions (a = 4, 5, 6; $F_{4\text{-}6}$). Its derivative gives rise to the field strength F, which in turn generates charge, potential gradients and recombination. These geometric effects are modelled using the (inertial) rotation of fions in the dimensional planes $D_{45/46/56}$.

The FSM metric (2.18) from point 1 is:

$$ds^2 \supset 2(A_\mu^a + \delta A_\mu^a)\, dy_a\, dx^\mu$$

A_μ^a generates the compact coordinates y_a for the visible dimensions x^μ.

- A_μ^a – off-diagonal component; undisturbed calibration potential; dimensionless in 7D; generates fields from the geometry
- dy_a – the dynamic length of the radius parallel to the unit vector for a = 4, 5, 6
- Group index a = 4, 5, 6; μ = 0, 1, 2, 3

The field strength F, which generates the interaction, arises from the covariant derivative of the gauge potential.

7D:

$$F^a_{MN} = \partial_M (A^a_N + \delta A^a_N) - \partial_N (A^a_M + \delta A^a_M) + g^{(7)}_M f^{abc} (A^b_M + \delta A^b_M)(A^c_N + \delta A^c_N) \quad (5.66)$$

4D (following compaction):

$$F^a_{\mu\nu} = \partial_\mu (A^a_\nu + \delta A^a_\nu) - \partial_\nu (A^a_\mu + \delta A^a_\mu) + g^{(4)} f^{abc} (A^b_\mu + \delta A^b_\mu) (A^c_\nu + \delta A^c_\nu) \quad (5.67)$$

- F – the field strength from potential A determines the resulting dynamics
- $\partial_\mu A^a_\nu - \partial_\nu A^a_\mu$ – Abelian part
- $g^{(4)} f^{abc} A^b_\mu A^c_\nu$ – non-Abelian part; gives rise to the self-interaction of the gauge bosons (fions)
- Indices: b, c = 4, 5, 6

Potential A must be determined so that conclusions can be drawn about the field forces.

Link to the particle model – rotation of the active fione:

Particles consist of spheres S containing active fions that oscillate in 4-dimensional rotational orbits. Rotations relative to the D_{56} dimensional plane generate a charge Q through a potential gradient parallel to the fourth dimension. **Axiom 11** attributes the universal electric potential to a displacement current resulting from the alignment of its photon field within the wave-field, accompanied by the simultaneous dynamic expansion of space-time.

$$A^a_\mu = \phi_a \, \delta^a_b \, \delta^c_\mu$$

- ϕ_a – scalar potential of an object's vibration in the compact wave field parallel to a direction a = 4, 5, 6; the amplitude is generated
- δ^a_b – Kronecker delta; (1, if $a = 6$; otherwise 0); $a = 6$ applies when the field exchange takes place in the D_{56} dimensional plane between the wave-field and the particle-field; otherwise, '0' → denotes invisible matter; b corresponds here to the dimensional plane in which the rotation in the wave-field takes place; for D_{45}: δ^a_4; for $D_{45/46}$: δ^a_5; for $D_{45/46/56}$: δ^a_6
- $\delta^c_\mu = \delta^0_\mu = 1$ only if $\mu = c = 0$ → contribution relating solely to the time component A^a_0
  electrical potential (charge effect) → $\mu = 0$
- $\delta^c_\mu = \delta^i_\mu = 1$ only if $\mu = c = i$ → contribution relating solely to the spatial component A^a_i
  magnetic/vector potential (motion/rotation effect) → $\mu = i$ = 1, 2, 3 or $\mu = 1$ for D_{14}, $\mu = 2$ for D_{24}, $\mu = 3$ for D_{34}; $i = D_{14/24/34}$

The potential A_μ^a is generated by the coupling between x^μ and y_a. The off-diagonal entries lie within the wave-field region of the metric. Compactification is achieved by integrating over the volume of the compact dimensions y_4, y_5, y_6. The volume integration ultimately yields the potential.

The effective action is:

$$S_{eff}^{(4)} = \frac{1}{V} \int \mathcal{L}_{(7)}\, d^7x = \int \mathcal{L}_{(4)}\, d^4x\, d^3y = \frac{1}{V} \int \mathcal{L}_{(4)}\, d^4x \sqrt{-g}\, d^3y$$

with: $\int d^3y = V = (2\pi R)^3$ und: $\mathcal{L}_{(4)}^{(\text{vectorial})} = -\frac{1}{4} g^{\mu\sigma} g^{\nu\rho} F_{\mu\nu}^a F_{\sigma\rho a} = -\frac{1}{4} 2\Delta A_\mu^a$

and: $\sqrt{-g} \rightarrow \sqrt{g_{aa}} \approx R \rightarrow A_\mu^{a(7)} = A_\mu^{a(4)} \sqrt{g_{aa}} = A_\mu^{a(4)} R$

- After the compactification, the following applies to the potential: $A_\mu^{a(4)} = \frac{A_\mu^{a(7)}}{R}$

Using $dy_a = R\,(\cos(kt+\beta))$ as the displacement length, the path for generating the potential in a direction '*a*' in the wave-field is described:

$$\int A_\mu^{a(4)}\, dy_a = \frac{A_\mu^{a(7)}}{R} R\,(\cos(kt+\beta)) = A_\mu^{a(7)} \cos(kt+\beta)$$

The Poisson equation gives:

$\nabla^2\phi_a = -\frac{\rho^a}{\epsilon_0}$; relevant is $a = 4$; time component with: $\mu = 0$; for electrical potential

The standard solution to Poisson's equation in flat space is:

$\rho^a = Q\, d^3(x)$

$$\phi_a = \frac{Q}{4\pi\, \epsilon_0\, R}$$

The FSM metric extends ϕ_a to $\Delta A_\mu^{a(7)}$ with $\phi_a\, dy_a\, \delta_b^a\, \delta_\mu^c$ to take account of the off-diagonal elements of the metric:

$$\Delta A_{eff,\,0}^a(x) = \frac{Q}{4\pi\, \epsilon_0\, R} \frac{1}{R} dy_a\, \delta_b^a\, \delta_\mu^c = \frac{Q}{4\pi\, \epsilon_0\, R} \cos(kt+\beta)\, \delta_4^a\, \delta_\mu^0 \qquad (2.68)$$

- ϕ_a – scalar potential of an object's vibration in the compact wave-field parallel to the fourth dimension; determines the amplitude
- $\cos(kt+\beta)$ – with: $\cos(kt = 0°) = 1$ generates the rotation of the fion, which touches the D_{56} dimensional plane at its greatest deflection.

These Kronecker deltas ensure the optimal case by ensuring that a measurable charge always occurs when cos(kt = 0°) = 1.

- (kt) – describes the characteristic repetition rate of a period
- β – deviation angle or phase shift; causes a possible shift in the geometric conditions for optimal field exchange in the D_{56} dimensional plane; acts in the same way as the gravitational component
- β = 0° → strong interaction
  $\beta = 0 < \beta < 90°$ → weak interaction
  β = 90° no longer interacts with the particle-field
- A_μ^a – now, after compactification, unit: $\frac{1}{\mathrm{m}}$ for standardisation using: ħ = c = 1, otherwise, electrodynamics in accordance with the unit: V
- Q – electric charge of the particle; unit: C (Coulomb) and e (elementary charge)
- R – Radius of the compact wave-field dimensions with $\lambda = 2\pi R$; unit: m; in the particle-field, the radius R is defined as the distance
- $\frac{Q}{4\pi\,\epsilon_0\,R}$ – classical Coulomb potential

<u>Potential gradient in the wave-field:</u>

During the recombination of the exchange fion with a passive fion to form an active fion, the potential difference ΔA_μ^a is generated. It persists for a brief moment until the polarisation of all active fions in the beam has been established. It therefore follows that:

$$\Delta A_\mu^a = (A_\mu^a + \delta A_\mu^a) - A_\mu^a = \delta A_\mu^a = \left(\frac{V_{rot,external}}{c} - \frac{V_{rot,bound}}{c}\right)\frac{Q}{4\pi\,\epsilon_0\,R}\cos(kt+\beta)\,\delta_4^a\,\delta_\mu^0$$

$V_{Rot} = \frac{c}{2}$ for bound (active) fions → $A_\mu^a\big|_{\text{bound}} = A_\mu^a + \delta A_\mu^a$

$V_{Rot} = c$ for unbound (exchange) fions → $A_\mu^a\big|_{\text{unbound}} = A_\mu^a$

a) for unbound (exchange) fions or visible light:

$$\Delta A_\mu^4 = \frac{c}{c}\,(1 - 1)\,\frac{Q}{4\pi\,\epsilon_0\,R}\cos(kt+\beta)\,\delta_4^a\,\delta_\mu^0 = 0$$

b) for bound (active) fions:

$$\Delta A_\mu^4 = \frac{c}{c}\,\left(1 - 1 + \frac{1}{2}\right)\frac{Q}{4\pi\,\epsilon_0\,R}\cos(kt+\beta)\,\delta_4^a\,\delta_\mu^0 = \frac{1}{2}\,\frac{Q}{4\pi\,\epsilon_0\,R}\cos(kt+\beta)\,\delta_4^a\,\delta_\mu^0$$

Taking the metric factor into account with 2:

$$2\Delta A_\mu^4 = 2\,\frac{1}{2}\,\frac{Q}{4\pi\,\epsilon_0\,R}\cos(kt+\beta)\,\delta_4^a\,\delta_\mu^0 = \frac{Q}{4\pi\,\epsilon_0\,R}\cos(kt+\beta)\,\delta_4^a\,\delta_\mu^0$$

- $A_\mu^a + \delta A_\mu^a$ – Potential difference relative to the reference potential; the superscript a denotes the compact wave-field dimensions; $a = 4$ for D_{45}, $a = 5$ for $D_{45/46}$, $a = 6$ for $D_{45/46/56}$; μ bottom for visible 4D space-time; $\mu = 0$ for time, $\mu = 1$ for D_{14}, $\mu = 2$ for D_{24}, $\mu = 3$ for D_{34}; $i = D_{14/24/34}$
- δA_μ^a – is non-zero in the short term → only a single registration is observed in the particle-field, like a single pulse or force peak; example: a fion is captured once in a beam
- δA_μ^a – is continuously non-zero → a constant interaction force is detected in the particle-field, which stabilises the bundle; example: strong interaction between two bosons (meson)
- $\frac{1}{2}$ – factor for recombination in the wave-field; it is **not** directly **recorded** as $\mathbf{\frac{1}{2}}$ **in the particle-field**. The metric factor 2, which takes the particle-field into account, fully compensates for the reset effect during binding.

<u>Taking object movement into account:</u>

An object movement V_3 causes a length contraction parallel to the dimensional plane D_{56}. The potential appears contracted and amplified relative to the resting position.

The electric potential for bound fions in a bundle is given by the formula:

$$A^a_{eff,\,0} = \frac{Q}{4\pi\,\epsilon_0\,R}\cos(kt+\beta)\,\delta_4^a\,\delta_\mu^0\,\frac{c}{V_5} \quad \text{with: } D_{56} \text{ – plane, } a = 6 \qquad (2.69)$$

The electric potential for unbound fions outside the bundle is given by:

$$A^a_{eff,\,0} = 0 \qquad (2.70)$$

- $\frac{c}{V_5}$ – field-enhancing effect caused by a contraction in length when an additional object moves within the particle-field in the direction of motion
- $V_5 = \sqrt{c^2 - v_3^2}$; V_3 – object velocity in a particle-field

<u>Specific potentials when an exchange fion recombines with visible matter in the ground state:</u>

$$\frac{c}{V_5} = \frac{c}{c} = 1;\ \delta_b^a = 1;\ \delta_\mu^c = 1;\ \cos(kt+\beta) = 1$$

a. An unbound exchange fion rotates freely in the dimensional plane D_{56}:

$$A_0^6 = \frac{e/3}{4\pi\,\epsilon_0\,R} = 0 \qquad \text{with: } (A_\mu^a + \delta A_\mu^a) - A_\mu^a = 0\text{, wegen } \delta A_\mu^a = 0 \tag{2.71}$$

b. Unbound exchange fion with an active fion:

$$A_0^4 = \frac{e/3}{4\pi\,\epsilon_0\,R} \qquad \text{with: } R = \frac{\lambda_{Obj}}{2\pi} \tag{2.72}$$

- a = 4 for recombination back into the dimensional plane D_{45}

c. A bound exchange fion with an electron (from a bundle):

$$A_0^5 = \frac{-\,e}{4\pi\,\epsilon_0\,R} \qquad \text{with: } R = \frac{\lambda_{Obj}}{2\pi} \tag{2.73}$$

- a = 5 for recombination into the possible dimensional planes $D_{45/46}$

d. A bound exchange fion with a positron:

$$A_0^5 = \frac{+\,e}{4\pi\,\epsilon_0\,R} \qquad \text{with: } R = \frac{\lambda_{Obj}}{2\pi} \tag{2.74}$$

- a = 5 for recombination into the possible dimensional planes $D_{45/46}$

e. A bound exchange fion with a positively charged boson:

$$A_0^6 = \frac{+\,e}{4\pi\,\epsilon_0\,R} \qquad \text{with: } R = \frac{\lambda_{Obj}}{2\pi} \tag{2.75}$$

- a = 6 for recombination into the possible dimensional planes $D_{45/46/56}$

The 7-dimensional FSM model explains how a potential arises in the wave-field. Most particles that mediate interactions must, whilst rotating, simultaneously possess a direction vector in the fourth spatial dimension. The rotation for a generation therefore takes place orthogonally to the D_{56} dimensional plane. In contrast, visible photons and neutrinos, for example, whose oscillation period runs exclusively parallel to the D_{56} dimensional plane, carry no potential.

9. Complete 7-dimensional field equations in FSM

Einstein's field equations describe curvature through energy, momentum, and matter. These field equations are extended with the field-space model, whereby the momentum-energy tensor T_{MN} receives contributions from the photon field, such as fions and relativistic fields of compact dimensions. The maximum velocity $V_{max} = c$

remains invariant, but the field propagation velocities V_4 and V_5 model the deformations. In this way, the 7-dimensional tensor can integrate orthogonal dimensional planes $D_{45/46/56}$ with causal causes such as mass from frequencies and charge from electric potential. The field equations in the FSM describe the unification of the four fundamental forces through geometric deformations in the wave-field. It also provides an explanation for dark energy and the expansion of the universe from its geometric rotation.

Initial situation - Definition of the Hilbert action in 7D of the FSM:

The derivation follows Hilbert's variational principle with $\delta S = 0$, where S is the total action

$S = S_g + S_m$ (Gravity + Matter),

which is extended to the FSM model.

The gravitational action according to Hilbert in 7D:

$$S_g = \frac{c^4}{16\pi G} \int R \sqrt{-g}\, d^7x = \frac{c^4}{16\pi G} \int \left(R_{MN} - \frac{1}{2} g_{MN} R\right) \delta g^{MN} \sqrt{-g}\, d^7x \tag{2.76}$$

- S_g – gravitational action; unit: Js; minimizes curvature; integrated over 7D space-time
- $\frac{c^4}{16\pi G}$ – standardized normalization constant; unit: $\frac{\text{kg}}{\text{m}^2\,\text{s}^2}$; scaled to field equations; determines the strength with which energy and momentum generate gravity
- $\int d^7x$ – integral over 7 dimensions; unit: m^7; the dimension of time is normalized to the maximum speed c in meters.
- $\sqrt{-g} = \sqrt{|\det(g_{MN})|}$ – determinant for the diagonal metric g_{MN}
- R – Ricci skalar from point 5.; unit: $\frac{1}{m^2}$; measures average curvature
- G – standardized gravitational constant; $G = 6{,}674\ 10^{-11}\ \text{N}\frac{\text{m}^5}{\text{kg}^2}$; the gravitational constant is diluted in the 7-dimensional field-space and only becomes concentrated through compactification; $G_{eff} = 6{,}674\ 10^{-11}\ \text{N}\frac{\text{m}^2}{\text{kg}^2}$;
- c – maximum speed; $c = 299792458\ \frac{\text{m}}{\text{s}}$

Matter action according to Hilbert in 7D: (photons/fions):

$$S_m = \int \mathcal{L}_m \sqrt{-g}\, d^7x = -\frac{1}{2} \int T_{MN}\, \delta g^{MN} \sqrt{-g}\, d^7x \tag{2.77}$$

The action minimizes the curvature with the Ricci scalar (R), taking into account the volume with: $\sqrt{-g}$. The FSM extends this with compact dimensions (M, N = 0,…, 6), which generate rotations parallel to the dimensional planes $D_{45/46/56}$, thus mechanically enabling the generation of a charge parallel to the electrical potential. d^7x is the integral over time, visible space, and compact dimensions.

- S_m – matter action; unit: Js; describes the dynamics of fields/particles
- $\mathcal{L}_m$– normalized matter Lagrangian density; unit: $\frac{\text{kg}}{\text{m}^2\,\text{s}^2}$; contains classical matter, calibration fields A^a_μ with potential terms and kinetic terms for the dynamic rotational mechanism in a vacuum
- $\delta S = 0$ – variation zero results in the stationary principle for equations

This step defines gravity geometrically and matter dynamically, whereby FSM integrates the matter action S_m through geometric fields.

Variation in total action according to the FSM metric:

$$\frac{\delta S}{\delta g^{MN}} = \frac{\delta S_g}{\delta g^{MN}} + \frac{\delta S_m}{\delta g^{MN}} = 0 \text{ leads to:}$$

From S_g:

$$\frac{\delta S_g}{\delta g^{MN}} = \frac{c^4}{16\pi G}\sqrt{-g}\,(R_{MN} - \frac{1}{2}\,g_{MN}\,R)$$

From S_m according to the formula (2.60):

$$\frac{\delta S_m}{\delta g^{MN}} = -\frac{1}{2}\sqrt{-g}\;T_{MN}$$

Overall:

$$\frac{c^4}{16\pi G}\sqrt{-g}\,(R_{MN} - \frac{1}{2}\,g_{MN}\,R) - \frac{1}{2}\sqrt{-g}\;T_{MN} = 0$$

$$R_{MN} - \frac{1}{2}\,g_{MN}\,R = \frac{8\pi G}{c^4}\,T_{MN} \tag{2.78}$$

with:

$$T_{MN} = T^{(\text{classical})}_{MN} + T^{(\text{global})}_{MN} + T^{(\text{Vector})}_{MN} \tag{2.65}$$

$$T_{MN} = [(\rho + \frac{p}{c^2})\,u_M\,u_N - p\,g_{MN}] + [\frac{c^4}{16\pi G}\,k^2_{Uni}\cos(k_{Uni}\,t)\,\eta_{MN}] + [\frac{1}{4\pi}\,(F^{\lambda a}_M\,F_{Na\lambda} - \frac{1}{4}\,g_{MN}\,F^a_{\rho\sigma}\,F^{a\rho\sigma})]$$

Global contribution:

$$G_{MN}^{(\text{global})} = R_{MN}^{(\text{global})} - \frac{1}{2}\,\eta_{MN}\,R^{(\text{global})} \tag{2.79}$$

- $R_{MN}^{(\text{global})} \approx \partial_P\,(-\frac{1}{2}\,k_{Uni}\,\sin(k_{Uni}\,t)\,\eta_N^P\,\delta_M^0) - \partial_N\,(-\frac{1}{2}\,k_{Uni}\,\sin(k_{Uni}\,t)\,\eta_P^P\,\delta_M^0)$
- $R^{(\text{global})} \approx \eta^{MN}\,[\partial_P\,(-\frac{1}{2}\,k_{Uni}\,\sin(k_{Uni}\,t)\,\eta_N^P\,\delta_M^0) - \partial_N\,(-\frac{1}{2}\,k_{Uni}\,\sin(k_{Uni}\,t)\,\eta_P^P\,\delta_M^0)]$

Local contribution:

$$G_{MN}^{(\text{local})} = R_{MN}^{(\text{local})} - \frac{1}{2}\,\eta_{MN}\,R^{(\text{local})} \tag{2.80}$$

- $R_{MN}^{(\text{local})} = \partial_P\,(\frac{1}{2}\,\frac{GM}{c^2\,r^2}\,(1 + \cos(kt + \beta))\,\delta_N^r\,\delta_M^t) - \partial_N\Gamma_{PM}^P$
- $\partial_N\Gamma_{PM}^P \approx 0$ for low-field, radial, $M = N = t$
- $R^{(\text{local})} \approx \eta^{MN}\,[\partial_P\,(\frac{1}{2}\,\frac{GM}{c^2\,r^2}\,(1 + \cos(kt + \beta))\,\delta_N^r\,\delta_M^t)]$

Vector contribution:

$$G_{MN}^{(\text{Vector})} = R_{MN}^{(\text{Vector})} - \frac{1}{2}\,\eta_{MN}\,R^{(\text{Vector})} \tag{2.81}$$

- $R_{MN}^{(\text{Vector})} = \partial_P\,(\frac{1}{2}\,\eta^{P\rho}\,\partial_M\,(A_\rho^a + \delta A_\rho^a)\,\delta_a^N)$
- $R^{(\text{Vector})} \approx \eta^{MN}\,[\partial_P\,(\frac{1}{2}\,\eta^{P\rho}\,\partial_M\,(A_\rho^a + \delta A_\rho^a)\,\delta_a^N)]$
- $\frac{\delta S_m}{\delta g^{MN}}$ – with $\frac{\delta}{\delta g^{MN}}$ as a functional variation by differentiation according to the contravariant metric; unit: $\frac{\text{kg}}{\text{m}^4\text{s}^2}$; measures how strongly matter reacts to a small change in the geometry (metric) of space-time – and that is precisely the local distribution of energy, momentum, and tension in space-time; indices *M, N* above are contravariant; *M*, *N* = 0,…, 6
- R_{MN} – Ricci tensor; unit: $\frac{1}{\text{m}^2}$; causes local curvature
- $\frac{1}{2}\,g_{MN}\,R$ – trace term; unit: $\frac{1}{\text{m}^2}$; makes Ricci component trace-free; g_{MN} from point 1.; *R*-scalar from point 5.
- $R_{MN} - \frac{1}{2}\,g_{MN}\,R$ – Einstein tensor according to the formula (2.51); unit: $\frac{1}{\text{m}^2}$; source of gravity; G_{MN} from point 6. without cosmological constant
- T_{MN} – impulse energy tensor; unit: $\frac{\text{kg}}{\text{m}^4\text{s}^2}$; source of matter; T_{MN} from point 7.
- $\frac{8\pi G}{c^4}$ – converted normalized coupling constant; unit: $\frac{\text{m}^2\,\text{s}^2}{\text{kg}}$; scales matter to curvature

- $\sqrt{-g}$ – invariant volume element; dimensionless normalized; measures how much the local volume is distorted by curvature compared to flat space

Leads to vacuum equations at $T = 0$, which are described in FSM by rotations.

<u>Replacing the cosmological constant Λ with $T_{MN}^{(\text{global})}$:</u>

Classically, a cosmological constant is added to account for vacuum energy. This is solved by the momentum-energy tensor T_{MN} of the FSM. The tensor T_{MN} has compact components with $T_{MN}^{(\text{global})}$ that describe the vacuum energy as a function of expansion with:

$$T_{MN}^{(\text{global})} = [\frac{c^4}{16\pi G} k_{Uni}^2 \cos(k_{Uni}\, t)\, \eta_{MN}]$$

The T_{MN} tensor only needs to be applied to the photon field of the universe.

Key finding:

The electromagnetic wave (moving energy) is identical to a dynamic deformation of space-time. It is not a wave in space-time, but rather space-time itself in wave-like motion. Gravity and electromagnetism are thus unified on the same geometric plane. Both are curvatures or rotations of the same 7-dimensional space-time.

If an object has a large mass, the classical term of the field equation (GTR) becomes dominant, whereas at the microscopic level, interaction forces cause space-time to curve due to their restoring forces.

10. Compactification from 7D to 4D with its principle of operation

Compactification is the process by which the three-dimensional invisible wave-field dimensions $F_{4\text{-}6}$ are integrated to obtain the effective four-dimensional observable world. In FSM, sinusoidal rotations stabilize compactification, generate effective fields, and explain masses and coupling frequencies as geometric effects. It resolves wave-particle duality and unifies the four fundamental forces by having compact modes generate particle masses.

<u>The starting point is the 7D metric according to (2.16) and separation of the dimensions:</u>

$$g_{MN} = [1 + \cos(k_{Uni}\, t)]\, (\eta_{MN} + h_{MN})$$

- g_{MN} – dimensionless

Minkowski-component η_{MN}:

- η_{MN} = diag(-1, 1, 1, 1, 1, 1, 1)

h_{MN} can be broke down into three blocks:

- Visible block: $h_{\mu\nu} = \frac{G\,M}{c^2\,r}\,(1+\cos(kt+\beta))\,\delta_{\mu\nu}$
- Coupling block: $h_{\mu m} = h_{m\mu} = (A_\mu^a + \delta A_\mu^a)\,\delta_{am}$; direction: μ

  $h_{m\nu} = h_{\nu m} = (A_\nu^a + \delta A_\nu^a)\,\delta_{am}$; direction: ν

- Compact block: diagonal, no additional scalar perturbation in the metric itself

Global contribution: $[1 + \cos(k_{Uni}\,t)]$

Indices: M, N = 0, 1, 2, 3, 4, 5, 6; μ, ν = 0, 1, 2, 3; a, m = 4, 5, 6

Reduction to an effective 4D metric:

$$ds^2 = g_{\mu\nu}^{4D}\,dx^\mu\,dx^\nu$$

$$g_{\mu\nu}^{4D} = [1 + \cos(k_{Uni}\,t)]\,[\eta_{\mu\nu} + \frac{G\,M}{c^2\,r}\,(1+\cos(kt+\beta))\,\delta_{\mu\nu}] \tag{2.82}$$

The compact dimensions vanish. Their influence is contained in the scalar fields $\phi_n(x)$ and the effective momentum-energy tensor $T_{\mu\nu}^{(\text{scalar})}$.

<u>Effective 4D field equations derived from compactification:</u>

The vacuum solution of the 7D Einstein tensor is:

$$G_{MN} = 0 = R_{MN}^{(7)} - \frac{1}{2}\,g_{MN}\,R^{(7)}$$

The action

$$S_g = \frac{c^4}{16\pi G}\int d^7x\,\sqrt{-g}\;R_{MN}$$

is integrated over the compact wavefield dimensions y^a with a = 4, 5, 6. This yields the invariant volume element of the compact dimensions V. The effective 4D action is:

$$S_g^{\text{eff}} = V\int\;(\sqrt{-g^{(4)}}\;(\frac{c^4}{16\pi G}\,R^{(4)} + \mathcal{L}_{\text{Matter}}^{\text{eff}})\;d^4x$$

- S_g^{eff} – effective gravitational force; unit: Js; minimises the curvature; results in observable equations
- V – compact volume; unit: m³; scales down from 7D to 4D
- $\int d^4x$ – 4D-integral; unity: m^4; creates visible space-time
- $\sqrt{-g^{(4)}}$ - 4D-volume element
- $R^{(4)}$ – 4D- scalar; unit: $\frac{1}{m^2}$; causes observable curvature
- $\mathcal{L}_{Matter}^{eff}$ – effective Lagrangian density for matter derived from the effective action S_g^{eff}
- $\frac{V}{G_{(7)}} = \frac{1}{G_{eff}}$ – The gravitational constant $G_{(7)}$, including compact wave-field dimensions, reduces to the following under compactification to G_{eff}

Variation of S_g^{eff} according to the reduced metric $g_{\mu\nu}^{4D}$:

$$\delta S_g^{eff} = \delta S_{EH}^{eff} + \delta S_{Matter}^{eff} = 0$$

Variation of the Einstein-Hilbert term (classical):

$$\delta S_{EH}^{eff} = \frac{c^4}{16\pi G_{eff}} \int (R_{\mu\nu}^{(4)} - \frac{1}{2} g_{\mu\nu}^{(4)} R^{(4)}) \delta g^{(4)\mu\nu} \sqrt{-g^{(4)}}\, d^4x + \text{scalar terms}$$

Variation in the amount of matter:

$$\delta S_{Matter}^{eff} = -\frac{1}{2} \int T_{\mu\nu}^{eff}\, \delta g^{(4)\mu\nu} \sqrt{-g^{(4)}}\, d^4x + \text{scalar terms}$$

This form first reduces the field equation from 7D to 4D. This yields the 4D gravitational component and a positive contribution from the gauge fields:

$$R_{\mu\nu}^{(4)} - \frac{1}{2} g_{\mu\nu}^{(4)} R^{(4)} = \frac{8\pi G_{eff}}{c^4} T_{\mu\nu}^{eff} \tag{2.83}$$

- $R_{\mu\nu}^{(4)}$ – effective 4D Ricci tensor; unit: $\frac{1}{m^2}$; observable curvature
- $g_{\mu\nu}^{(4)}$ – 4D-metric
- $R^{(4)}$ – Ricci scalar, 4D; unit: $\frac{1}{m^2}$; average curvature
- G_{eff} – Gravitational constant; unit: $6{,}674\ 10^{-11}\ N\frac{m^2}{kg^2}$;
- $T_{\mu\nu}^{eff}$ – effective momentum-energy tensor; unit: $\frac{J}{m^3} = \frac{kg}{m\, s^2}$; generates contributions from $F_{\mu\nu}^a$

Equation (2.65) reproduces the 4D Einstein-Yang-Mills equations with FSM-specific terms for the momentum-energy tensor $T_{\mu\nu}^{eff}$:

$T^{eff}_{\mu\nu} = T^{(\text{classical})}_{\mu\nu} + T^{(\text{global})}_{\mu\nu} + T^{(\text{Vector})}_{\mu\nu} + T^{(\text{scalar})}_{\mu\nu}$

$$T^{eff}_{\mu\nu} = [(\rho + \frac{p}{c^2})\, u_\mu\, u_\nu - p\, g^{(4)}_{\mu\nu}] + [\frac{c^4}{16\pi G_{eff}}\, k^2_{Uni} \cos(k_{Uni}\, t)\, \eta_{\mu\nu}] + [\frac{1}{4\pi}\, (F^{\lambda a}_{\mu}\, F_{\nu a\lambda} - \frac{1}{4}\, g^{(4)}_{\mu\nu}\, F^{a}_{\rho\sigma}\, F^{a\rho\sigma})] + T^{(\text{scalar})}_{\mu\nu} \quad (2.84)$$

Definition of the mode terms:

This step reduces 7D to 4D and generates Kaluza-Klein modes. The scalar field in 7D is expanded as a Fourier series in the compact coordinates:

$$\phi(x, y) = \phi_4 + \sum_{n=1}^{3} \phi_n(x) e^{\frac{iny^a}{R}} \quad (2.85)$$

- $\phi(x, y)$ – Scalar field; e.g., in the case of metric distortion; unit variable; generally produces a field in 7D
- $\phi_4(x)$ – n = 0–mode; global cosmic background potential parallel to the fourth dimension; unity: $\frac{1}{\text{m}}$
- $\phi_n(x)$ – 4D mode field; creates an effective 4D field; index n corresponds to the mode number for the fermion generation; unit: $\frac{1}{\text{m}}$
- $e^{\frac{iny^a}{R}}$ – periodic function; results in a harmonic expansion; i represents the imaginary unit; magnitude is always 1
- y^a – compact coordinate; unit: m; is "rolled up"; is invisible
- R – radius of the compact wave-field dimensions with $\lambda = 2\pi R$; unit: m; is the scale of compactification; if small → invisible

After compaction:

$$\phi(x) = \sum_{n=1}^{3} \phi_n(x) e^{\frac{iny^a}{R}} \quad (2.86)$$

The complex exponential is taken in the real domain; standing wave as a cosine or sine function.

Possible scale terms and stabilization of compaction:

Scalar terms contribute to the stabilization of the compactification. They generate a restoring force that acts in proportion to a potential field ϕ_n to keep the field centered at the origin. They also carry an effective 4-dimensional scalar field ϕ_n, which represents the nth mode excitation of the wave in the compact wave-field dimensions. ϕ_n arises from the compactification of the three compact wave-field dimensions and the wave dynamics prevailing within them. This wave is described as a function of time t and the compact coordinates y_a.

$\phi(x^\mu, y^a)$ with: x^μ – 4D-coordinates; y^a – 3D-compact wave-field dimensions

Compact Fourier series / mode development:

$\phi(x, y^a + 2\pi R) = \phi(x, y^a)$

This allows for a Fourier expansion (Kaluza–Klein modes):

$\phi(x, y^a) = \sum_{n=-\infty}^{\infty} \phi_n(x)\, e^{\frac{iny^a}{R}}$

In its actual form:

a) $\phi(x, y^a) = \sum_{n=1}^{\infty} \phi_n(x) \sin(\frac{ny^a}{R})$ or:

b) $\phi(x, y^a) = \sum_{n=1}^{\infty} \phi_n(x) \cos(\frac{ny^a}{R})$ (2.87)

Option b) is chosen because it allows for a phase shift of the potential relative to the D_{56} plane. This phase shift transforms the strong interaction into a weak interaction.

A) Mass component $V(\phi_n)_m$:

$\phi(x, y^a) = \sum_{n=1}^{\infty} \phi_n(x) \cos(\frac{ny^a}{R})$

The wave equation for compact dimensions:

$(\partial_\mu \partial^\mu + \partial_a \partial^a)\, \phi_n = 0$ (2.88)

$(\partial_\mu \partial^\mu)\, \phi_n = 0$ → the propagation of a massless scalar field does not involve mass

In a linear approximation:

$\partial_t^2 \phi_n + \partial_a \partial^a \phi_n = 0$

$\partial^a \phi_n \sim \frac{n}{R} \phi_n$ (Derivation with respect to y^a provides the factor $\frac{n}{R}$)

$\partial_a \partial^a \phi_n \sim -(\frac{n}{R})^2 \phi_n$ (second derivative with respect to y^a)

$\partial_t^2 \phi_n \pm (\frac{n}{R})^2 \phi_n = 0$

Similar to:

$\partial_t^2 \phi_n + \frac{\partial V}{\partial \phi_n} = 0$

Therefore:

$\frac{\partial V}{\partial \phi_n} = (\frac{n}{R})^2 \phi_n$

Integration with respect to ϕ_n provides:

$$V(\phi_n) \supset V(\phi_n)_m = \int \left(\frac{n}{R}\right)^2 \phi_n \, d\phi_n = \frac{1}{2}\left(\frac{n}{R}\right)^2 \phi_n^2 + \text{constant}$$

$$\boldsymbol{V(\phi_n)_m = \frac{1}{2}\left(\frac{n}{R}\right)^2 \phi_n^2} \qquad (2.89)$$

- $\frac{1}{2}\left(\frac{n}{R}\right)^2 \phi_n^2$ – unit: $\frac{1}{\text{m}^2}\frac{1}{\text{m}^2} = \frac{1}{\text{m}^4}$; n = dimensionless; $[R]$ = m; scaled by $c\hbar$
- $\phi_n(x)$ – nth-generation scalar field (n = 1 light, n = 2 medium, n = 3 heavy)
- $\phi_n(x)$ – becomes a single unit after compactification in 4D: $\frac{1}{\text{m}}$; is derived from the wave amplitude in the wave-field dimensions
- $\phi_n(x)$ – contributes to mass, chirality, and possible interactions

With the visible modes n = 1, 2, 3, $\phi(x, y^a)$ becomes $\phi(x)$.

Renormalization using $c\hbar \frac{1}{c^2}$, due to the time component $\mu = 0$, provides the mass of an object, which depends on its geometry:

$$\boldsymbol{m = \frac{n\,h}{2\pi\, c\, R}} \qquad (2.90)$$

Geometrically dependent mass with its amplitude and as a periodic disturbance:

$$m(t) = \frac{n\,h}{2\pi\, c\, R} \cos(kt + \beta) \qquad (2.91)$$

Relativistic without periodic perturbation with $\cos(kt = 0° + \beta = 0°)$:

The wavelength $\lambda = 2\pi R$ causes a contraction in the direction of motion, resulting in what is known as a blue shift at the source. In the case of a blue shift, the contraction term in the numerator is multiplied. This explains the conversion of kinetic energy into rest mass, whereby an object's velocity contracts the field propagation velocity V_5 from its resting state.

$\lambda(t) = \lambda \sin(\alpha) = \lambda \frac{V_5}{c}$ → The wavelength decreases as the object's velocity V_4 increases

Note: Blue shift is never observed from the outside, even when the source is moving parallel to the observer, because the observer always detects a lengthening of the observed wavelength between the source and the observer.

Insert:

$$m(t) = \frac{n\,h}{c\,\lambda}\,\frac{1}{\sin(\alpha)} = \frac{n\,h}{c\,\lambda}\,\frac{c}{V_5} \qquad \text{(Wavelength with blue shift)} \qquad (2.92)$$

B) Dynamic, oscillating component $V(\phi_n)$ for coupled and uncoupled matter:

The metric component that describes global and local volume modulation is:

$\gamma(t) = 1 + \cos(kt)$

$$R^{(4)}_{\mu\nu} - \frac{1}{2}\, g^{(4)}_{\mu\nu}\, R^{(4)} = \frac{8\pi G}{c^4}\, T^{(\text{global})}_{MN} \int d^3y\, \sqrt{\gamma} \qquad \text{with: } \sqrt{\gamma} = \sqrt{\det(\gamma)}$$

$\gamma(t) = (1 + \cos(kt))$

Potency: 3 due to 3D Euclidean

y^4, y^5, y^6

$$\sqrt{\det(\gamma)} = \sqrt{(1 + \cos(kt))^3} = (1 + 3\cos(kt) + 3\cos^2(kt) + \cos^3(kt))^{1/2}$$

$$\sqrt{\det(\gamma)} \approx 1 + \frac{3}{2}\cos(kt) \qquad \text{(after linear approximation)}$$

The effective scalar field $\phi_n(x)$ is defined such that it is proportional to the relative change in volume:

$$1 + \frac{3}{2}\cos(kt) = 1 + \phi_n(x) \qquad \rightarrow \qquad \phi_n(x) = \frac{3}{2}\cos(kt + \text{phase})$$

written in simple notation: $\phi_n(x) = \phi_n$

Effective energy density due to coupling effects (interactions):

- First derivative (velocity) of the potential:
  $$\frac{dV}{dt} = \partial_t\phi_n = -\frac{3}{2}\,k\sin(kt + \text{phase})$$
- The kinetic energy density for the 4D Lagrangian density is proportional to:
  $$E_{\text{kin}} = \frac{1}{2}\,(\partial_t\phi_n)^2 = \frac{1}{2}\left(-\frac{3}{2}\,k\sin(kt + \text{phase})\right)^2 = \frac{9}{8}\,k^2\sin^2(kt + \text{phase})$$
- The second derivative as the acceleration of the field:
  $$\partial_t^{\,2}\phi_n = -\frac{3}{2}\,k^2\cos(kt + \text{phase})$$
- The acceleration of the change in volume contributes to the effective potential energy density: $E_{pot} \sim (\text{acceleration})^2$ oder $\sim (\text{elongation})^2$

$$E_{pot} = (\partial_t^2 \phi_n)^2 = (-\frac{3}{2} k^2 \cos(kt + \text{phase}))^2 = \frac{9}{4} k^4 \cos^2(kt + \text{phase})$$

The potential energy density E_{pot} is equal to $V(\phi_n)_{pot}$:

$$V(\phi_n) \supset V(\phi_n)_{pot} = \frac{9}{4} k^4 \cos^2(kt + \text{phase}) = \frac{9}{4} k^4 \cos^2(kt + \text{phase})$$

Potential $V(\phi_n)_{pot}$ for coupled matter:

$$\boldsymbol{V(\phi_n)_{pot} = \frac{9}{4} k^4 \cos^2(kt + \beta)} \qquad (2.93)$$

- $V(\phi_n)_{pot}$ – potential energy density for coupled matter; unit: $\frac{J}{m^3} = \frac{kg}{m\,s^3}$; scaled by $c\hbar$; means: energy becomes accessible and is coupled
- ϕ_n lies in the geometric change in the oscillation with:
  $\phi_n = \frac{3}{2} \cos(kt + \text{phase})$
- (kt) – Due to its periodic change in direction, the curve of the cosine function repeatedly restores the dynamics of space-time, so that it does not lead to a collapse—neither in contraction nor in expansion.
- β – angles of deviation that shift the phase relative to the dimensional plane D_{56}
  global: $\beta = 0$, local: $\beta = 0 \ldots 90°$
- k – angular frequency of the oscillation; unit: $\frac{1}{s}$
- k^4 – unit: $\frac{1}{s^4}$; due to the metric normalization of time, k^4 is interpreted as: $\frac{1}{m^4}$; energy density is calculated by multiplying by: $\frac{kg}{m\,s^2}$
- Energy is proportional to the square of the amplitude → $\cos^2(kt + \beta)$, enables a resonance or phase shift that prevents the system from reaching an unstable fixed point

Potential $V(\phi_n)_{dark}$ for decoupled matter (dark energy):

As long as the universe remains below the wavelength λ_{min}, which prevents quantized photons from interacting electrically, the minimum coupling frequency $f < f_{min}$ is not yet reached. In this state, the photon field consists exclusively of dark energy. Upon exceeding the minimum coupling frequency f_{min}, the potential $V(\phi_n)_{dark}$ is converted into $V(\phi_n)_{pot}$ over the entire expansion phase. The two potentials are thus complementary to one another.

$$\mathcal{L}_n \supset V(\phi_n)_{dark} = \frac{9}{4} k^4 (1 - \cos(kt + \beta))^2 \qquad (2.94)$$

- $V(\phi_n)_{dark}$ – energy density for dark energy; unit: $\frac{J}{m^3} = \frac{kg}{m\,s^3}$; scaled by $c\hbar$; “still available, but not accessible”; does not interact with bound matter
- $(1 - \cos(kt + \beta))^2$ – The square term resulting from the cosine oscillation with phase β makes the term oscillatory and dynamic; the energy density is positive and the displacement becomes quadratic (by comparison, the cosmological constant Λ from classical literature is, in contrast, merely a fixed constant)

$$\boldsymbol{V(\phi_n)_{dark} = \frac{9}{4}\, k^4\, (1 - \cos(kt + \beta))^2} \tag{2.95}$$

Stabilization via scalar potential $V(\phi_n)$:

$V(\phi_n) = V(\phi_n)_m + V(\phi_n)_{pot} + V(\phi_n)_{dark}$

$$V(\phi_n) = \frac{1}{2}\left(\frac{n}{R}\right)^2 \phi_n^2 + \frac{9}{4}\, k^4 \cos^2(kt + \beta) + \frac{9}{4}\, k^4\, (1 - \cos(kt + \beta))^2 \tag{2.96}$$

- $\frac{1}{2}\left(\frac{n}{R}\right)^2 \phi_n^2$ – Mass term for the scalar field ϕ_n. It ensures that ϕ_n oscillates around $\phi_n = 0$ and cannot become arbitrarily large. This corresponds to a positive curvature in the potential, which binds the field to the origin.
- $\frac{9}{4}\, k^4 \cos^2(kt + \beta)$ – oscillating periodic term. It creates a dynamic, time-dependent barrier in the potential. Through the phase shift $(kt + \beta)$, the scalar field ϕ_n actively couples to the oscillation of the compactification via (kt). This prevents the compact volume (or radius R) from permanently collapsing or growing exponentially; instead, it is periodically pushed back.

<u>Composition of the scalar terms for the momentum-energy tensor $T_{\mu\nu}^{(scalar)}$:</u>

The Lagrangian density per mode n is, finally:

$$\mathcal{L}_n = \frac{1}{2}\, \partial_\mu \phi_n\, \partial^\mu \phi_n - [(V(\phi_n)_m + V(\phi_n)_{pot} + V(\phi_n)_{dark})]$$

$$S_{scalar} = \int d^4x \sqrt{-g^{(4)}}\, \sum_{n=1}^{3} \mathcal{L}_n$$

The components of the momentum-energy tensor are defined as:

$$T_{\mu\nu}^{(\mathrm{scalar})} = -\frac{2}{\sqrt{-g^{(4)}}}\, \frac{\delta S_{scalar}}{\delta g^{\mu\nu}}$$

a) Variation of the kinetic term:

$$\mathcal{L}_{kin} = \frac{1}{2}\,\partial^{\mu}\,\phi_n\,\partial_{\mu}\,\phi_n = \frac{1}{2}\,g^{\mu\rho}\,\partial_{\mu}\,\phi_n\,\partial_{\rho}\,\phi_n$$

→ this term depends explicitly on $g^{\mu\nu}$ ab; direct variation $\neq 0$

Variation, according to $g^{\mu\nu}$:

- Direct variation: $\frac{1}{2}\,\partial_{\mu}\,\phi_n\,\partial_{\nu}\,\phi_n$
- Metrical variation: $-\frac{1}{2}\,g^{(4)}_{\mu\nu}\,\frac{1}{2}\,\partial^{\rho}\,\phi_n\,\partial_{\rho}\,\phi_n$

is typically, in the Lagrangian density of scalar fields, the kinetic trace term arising from the variation of $\sqrt{-g^{(4)}}$ and $g^{\mu\rho}$.

$$\frac{\delta\mathcal{L}_{kin}}{\delta g^{\mu\nu}} = \frac{1}{2}\,\partial_{\mu}\,\phi_n\,\partial_{\nu}\,\phi_n - \frac{1}{2}\,g^{(4)}_{\mu\nu}\,\frac{1}{2}\,\partial^{\rho}\,\phi_n\,\partial_{\rho}\,\phi_n$$

b) Variation of the mass term:

$$\mathcal{L}_{Mass} = V(\phi_n)_m$$

$$\frac{\delta\mathcal{L}_{Mass}}{\delta g^{\mu\nu}} = -\frac{1}{2}\,g^{(4)}_{\mu\nu}\,V(\phi_n)_m$$

c) Variation in the potential for coupled matter:

$$\mathcal{L}_{pot} = V(\phi_n)_{pot}$$

$$\frac{\delta\mathcal{L}_{pot}}{\delta g^{\mu\nu}} = -\frac{1}{2}\,g^{(4)}_{\mu\nu}\,V(\phi_n)_{pot}$$

d) Variation in the potential for uncoupled matter:

$$\mathcal{L}_{dark} = V(\phi_n)_{dark}$$

$$\frac{\delta\mathcal{L}_{dark}}{\delta g^{\mu\nu}} = -\frac{1}{2}\,g^{(4)}_{\mu\nu}\,V(\phi_n)_{dark}$$

e) Variation of the volume element $\sqrt{-g^{(4)}}$:

$$\delta(\sqrt{-g^{(4)}}\,\mathcal{L}_n) = \frac{1}{2}\,g^{(4)}_{\mu\nu}\,\sqrt{-g^{(4)}}\,\mathcal{L}_n\,\delta g^{\mu\nu} + \sqrt{-g^{(4)}}\,\frac{\delta\mathcal{L}_n}{\delta g^{\mu\nu}}\,\delta g^{\mu\nu}$$

In summary:

$$\frac{\delta S_{scalar}}{\delta g^{\mu\nu}} = \sqrt{-g^{(4)}}\,[\frac{1}{2}\,\partial_\mu\,\phi_n\,\partial_\nu\,\phi_n - \frac{1}{2}\,g^{(4)}_{\mu\nu}\,(\frac{1}{2}\,\partial^\rho\,\phi_n\,\partial_\rho\,\phi_n + V(\phi_n)_m + V(\phi_n)_{pot} + V(\phi_n)_{dark})]$$

$$T^{(scalar)}_{\mu\nu} = -\frac{2}{\sqrt{-g^{(4)}}}\,\frac{\delta S_{scalar}}{\delta g^{\mu\nu}}$$

(the negative sign has already been included in the variations) (2.97)

$$T^{(scalar)}_{\mu\nu} = c\hbar\,\sum_{n=1}^{3}\,[\partial_\mu\,\phi_n\,\partial_\nu\,\phi_n - \frac{1}{2}\,g^{(4)}_{\mu\nu}\,(\partial^\rho\,\phi_n\,\partial_\rho\,\phi_n + 2V(\phi_n)_m + 2V(\phi_n)_{pot} + 2V(\phi_n)_{dark})]$$

- $T^{(scalar)}_{\mu\nu}$ – scalar energy density; unit: $\frac{J}{m^3} = \frac{kg}{m\,s^3}$, scaled by $c\hbar$; The scalar tensor contributes to the curvature in Einstein's field equations and, in the FSM, accounts for the masses of the generations with $(\frac{n}{R})^2$ and possible dynamic energy density $V(\phi_n)$.
- $\sum_{n=1}^{3}$ – Sum over the three modes $n = 1, 2, 3$
- $\partial_\mu\,\phi_n$ or $\partial_\nu\,\phi_n$ – covariance gradient (directional derivative) of the field ϕ_n
- $\partial_\mu\,\phi_n\,\partial_\nu\,\phi_n$ – directional **kinetic momentum contribution** (gradient product); $\mu, \nu = 0, 1, 2, 3$; 0 = time, 1, 2, 3 space; describes the directed flow of energy and momentum in the scalar field ϕ_n; unit: $\frac{1}{m^4}$, $[\phi_n] = \frac{1}{m}$ → after normalization: $\frac{kg}{m\,s^3}$; scaled by $c\hbar$
- $\frac{1}{2}$ – Normalization factor derived from the variation in effect; ensures the correct balance between the kinetic and potential contributions
- $g^{(4)}_{\mu\nu}$ – effective 4D metric after reduction; dimensionless; defines distances and curvature in the observable 4D world
- $\partial^\rho\,\phi_n\,\partial_\rho\,\phi_n$ – **isotropic contribution of kinetic pressure** (contracted gradient); $\rho = 0, 1, 2, 3$; determines the total kinetic energy density regardless of direction; unit: $\frac{1}{m^4}$, $[\phi_n] = \frac{1}{m}$ → after normalization: $\frac{kg}{m\,s^3}$, scaled by $c\hbar$
- $V(\phi_n)$ – terms for **potential energy**; unit: $\frac{1}{m^4}$, $[\phi_n] = \frac{1}{m}$ → after normalization: $\frac{kg}{m\,s^3}$; scaled by $c\hbar$
- $V(\phi_n)_m$ – mass term resulting from compactification; $n = 1, 2, 3$; $[R] = m$; $[\phi_n] = \frac{1}{m}$; causes the field ϕ_n to have an effective mass $m_n = \frac{n}{R}$ obtained; scaled by $c\hbar$; note the generational hierarchy
- $V(\phi_n)_{pot}$ – potential of the scalar field ϕ_n (oscillating and stabilizing); potential energy; scaled by $c\hbar$

- $V(\phi_n)_{dark}$ – The potential of the scalar field of uncoupled matter from the dark energy sector; scaled by $c\hbar$; $V(\phi_n)_{dark}$ and $V(\phi_n)_{pot}$ act in a complementary manner

Notes:

- $n = 1$; ϕ_1; Ground state (lightest field); lightest generation; e.g., electron, up and down quarks (u/d-quarks)
- $n = 2$; ϕ_2; first excited state; second generation; e.g., muons, C-quarks, S-quarks
- $n = 3$; ϕ_3; second excited state; third generation; e.g., tauon, B-quark, T-quark
- **Casimir energy** (quantum fluctuations) in compact spaces generates gauge fields or scalar fields with energy that depends on the radius:
  $V_{Casimir} \sim \frac{hc}{2\pi R^4}$
  In FSM, this is integrated using the potential of the scalar field $V(\phi_n)$ and modeled as an oscillating term, since the time specification $\sim [k^4] = \frac{1}{s^4}$ is normalized by the spatial length $[t\ c]$ = m for 4D.
- The mass term: $V(\phi_n)_m = \frac{1}{2}\left(\frac{n}{R}\right)^2 \phi_n^2$ corresponds to a Kaluza-Klein mass with $m_n = \frac{n}{R}$. The radius implies that its size depends on any electromagnetic oscillation of an object in space-time. The smaller the radius, the shorter its wavelength, the higher its frequency, the greater the inertial motion of the oscillation in space-time, and the greater its explicit mass. This term also explains why **masses** are hierarchical ($n = 1$, $n = 2, 3$ heavier) (classified as "families of dimensions" in **Chapter 3.5**), why there are exactly three generations ($n = 1, 2, 3$), and why the smallest boson masses exist only for $n = 1, 2$ in the dimensional plane $D_{45/46}$. The **potential $V(\phi)$ stabilizes** the system. The **electron**/positron is the **fundamental particle** of mode n = 1, and, as will be shown, the first particle to interact electrically with photons from the dark energy sector once the minimum coupling frequency fmin is exceeded. Thus, the **Higgs field theory** is replaced by the FSM model.

<u>Effective momentum-energy tensor $T_{\mu\nu}^{eff}$ including scale terms:</u>

$$T_{\mu\nu}^{eff} = T_{\mu\nu}^{(\text{classical})} + T_{\mu\nu}^{(\text{global})} + T_{\mu\nu}^{(\text{Vector})} + T_{\mu\nu}^{(\text{scalar})}$$

$$T_{\mu\nu}^{eff} = [(\rho + \tfrac{p}{c^2})\, u_\mu\, u_\nu - p\, g_{\mu\nu}^{(4)}] + [\tfrac{c^4}{16\pi G_{eff}}\, k_{Uni}^2 \cos(k_{Uni}\, t)\, \eta_{\mu\nu}] + [\tfrac{1}{4\pi}\, (F_\mu^{\lambda a}\, F_{\nu a\lambda} - \tfrac{1}{4}\, g_{\mu\nu}^{(4)}\, F_{\rho\sigma}^{a}\, F^{a\rho\sigma})] +$$
$$c\hbar \textstyle\sum_{n=1}^{3} [\partial_\mu\, \phi_n\, \partial_\nu\, \phi_n - \tfrac{1}{2}\, g_{\mu\nu}^{(4)}\, (\partial^\rho\, \phi_n\, \partial_\rho\, \phi_n + 2V(\phi_n)_m + 2V(\phi_n)_{pot} + 2V(\phi_n)_{dark})] \quad (2.98)$$

- $V(\phi_n)_m = \frac{1}{2}\left(\frac{n}{R}\right)^2 \phi_n^2$
- $V(\phi_n)_{pot} = \frac{9}{4}\, k^4\, (\cos(kt + \beta))^2$

- $V(\phi_n)_{dark} = \frac{9}{4} k^4 (1 - \cos(kt + \beta))^2$
- $V(\phi_n)_{pot}$ and $V(\phi_n)_{dark}$ work in complement to each other
- Scaling terms are scaled by $c\hbar$
- The potentials have the same impact locally and globally, depending on the context.

The ratio of matter that couples to matter that does not yet couple:

$$\frac{\rho_{coupled}}{\rho_{dark}} = \frac{V(\phi_n)}{V(\phi_n)_{dark}} = \frac{(\cos(90° - \alpha))^2}{(1 - \cos(90° - \alpha))^2} = \frac{(\sin(\alpha))^2}{(1-\sin(\alpha))^2} \quad (2.99)$$

- α – trigonometric solid angle at the current point in space-time; angle between the global field velocities V_4 and V_5 (cathetus) of the universe relative to the maximum velocity c (hypotenuse)
- Neglecting particles with weak interactions with β
- Extreme values, taking into account their complementary effects:
  0 – stands for 100% dark energy
  ∞ – refers to the conversion of dark energy into 100% coupled matter
- Example: Expansion of the universe with $kt = \alpha = 18{,}7°$ (from **Chapter 7.2**):
  a) cos(90° - 18,7°) = 0,32; corresponds to the coupled matter; with starting point cos(90°) = 0 proportions of bound matter
  b) 1 - cos(90° - 18,7°) = 0,68; is consistent with astronomical observations of dark energy

Derivation of Relativistic Momentum and Energy from FSM Geometry:

To extract the energy from T_{00} and the momentum from T_{0i}, the effective tensor, denoted by $T^{eff}_{\mu\nu}$, is already given in compact form.

$$T^{(\text{classical})}_{\mu\nu} = (\rho + \frac{p}{c^2})\, u_\mu\, u_\nu - p\, g^{(4)}_{\mu\nu} \qquad \text{Normalization: } G_{\mu\nu}\, u^\mu\, u^\nu = - c^2$$

Applying the indices results in:

a) T_{00} – energy density; energy flux in the timelike direction; 3D volume element

b) T_{i0} – energy flux density; energy flux in the spatial i-direction; $i = 1, 2, 3$; 3D-volume element: 1x timelike, 2x spatial axes

c) T_{0i} – Pulse density; pulse flux in the timelike direction; 3D volume element

d) T_{ik} – Pulse current density; pulse flux of the kth component of the pulse in the spatial i-direction; 3D volume element: 1x timelike, 2x spatial axes

a) Energy density with $p = 0$; $u_0 \approx - c$; $u_i \approx 0$ the rest mass dominates:

$T_{00} \approx \rho\, c^2$; $T_{0i} \approx 0$; $T_{ij} \approx 0$

Relativistic:

$$\boldsymbol{T_{00} \approx \rho\, c^2 \frac{1}{\sin^2(kt)}} \qquad (2.100)$$

The integral over the volume V yields:

$$E = \int\, T_{00}\, \mathrm{d}V = \int\, \rho\, c^2 \frac{1}{\sin^2(kt)} \sin(kt)\, \mathrm{d}V = \rho\, c^2 \frac{V}{\sin(kt)}$$

$$\boldsymbol{E(t) = m\, c^2 \frac{1}{\sin(kt)}} \qquad \text{with: } \frac{1}{\sin(kt)} = \frac{c}{V_5} \qquad (2.101)$$

For $p \neq 0$:

$$T^{00} = (\rho\, c^2 + p)\, c^2 \frac{1}{c^2} - p$$

Relativistic:

$$\boldsymbol{T^{00} = (\rho\, c^2 + p) \frac{1}{\sin^2(kt)} - p} \qquad (2.102)$$

c) Pulse density with $p \approx 0$; $u_0 \approx - c$; $u_i \approx \frac{V_i}{c} << 1$ ($\frac{1}{c}$ follows from the metric normalization of $\eta_{\mu\nu}$ for one of the three spatial directions)

$$T_{0i} \approx - \rho\, c \frac{V_i}{c}$$

In the literature, contravariant indices are used, which makes the expression positive:

$$T^{0i} \approx \rho\, V_i$$

Relativistic with: $\frac{1}{\sin(kt)} = \frac{c}{V_5}$

$$\boldsymbol{T^{0i} \approx \rho\, V_i\, \frac{1}{\sin^2(kt)}} \qquad (2.103)$$

The integral over the volume V provides:

$$p_i = \int\, T^{0i}\, \mathrm{d}V = \int\, \rho\, V_i \frac{1}{\sin^2(kt)} \sin(kt)\, \mathrm{d}V = \rho\, V_i \frac{V}{\sin(kt)}$$

$$\boldsymbol{p_i(t) = m\, V_i \frac{1}{\sin(kt)}}$$ classical impulse

$$p(t) = m\, c \frac{1}{\sin(kt)} \quad \text{for } V_i \approx c \tag{2.104}$$

For $p \neq 0$:

$$T^{0i} = (\rho\, c^2 + p)\, V_i \frac{1}{c^2}$$

Relativistic:

$$\boldsymbol{T^{0i} = (\rho\, c^2 + p)\, V_i \frac{1}{c^2 \sin^2(kt)}} = (\rho + p)\, V_i \frac{1}{V_5{}^2} \tag{2.105}$$

- T_{00} – time-time extraction; unit: $\frac{kg}{m\,s^2}$; energy density as the primary source of gravity
- $p\, c^2$ – rest energy density; unit: $\frac{kg}{m\,s^2}$; increases effective mass/energy through field resistance or rotation in the wave-field parallel to the dimension D_4
- ρ – static density; unit: $\frac{kg}{m^3}$; mass per unit volume
- T_{0i} – space-time extraction; unit: $\frac{kg}{m^2\,s}$; describes the flow of momentum through a surface (e.g., the transfer of energy)
- - $\rho\, V_i$ – commonly referred to as pulse density or mass flow density; unit: $\frac{kg}{m^2\,s}$; results in momentum per unit area at a speed of V_i in direction i
- /sin(kt) – relativistic factor; increases momentum and energy through geometric dilation during rotation in compact dimensions
- $\eta_{\mu\nu}$ – metric; $\eta_{\mu\nu}$ = diag(-1,1,1,1)
- u_0 – index „0“ means, based on the metric $\eta_{\mu\nu}$ the time field; this cell contains – c, because the time was standardized for unity with [$c\, t$] = m; unit: [u_0] = $\frac{m}{s}$
- u_i – index „i“ denotes a direction derived from the metric $\eta_{\mu\nu}$, where the indices 1, 2, and 3 correspond to the 3D space; in these cells, the three spatial directions are normalized by $\frac{V_i}{c}$; dimensionless

<u>Global oscillating term:</u>

$$T_{\mu\nu}^{(\text{global})} = \frac{c^4}{16\pi G_{eff}} k_{Uni}^2 \cos(k_{Uni}\, t)\, \eta_{\mu\nu}$$

$$\text{a) } \boldsymbol{T_{00}^{(\text{global})}} = \frac{c^4}{16\pi G_{eff}} k_{Uni}^2 \cos(k_{Uni}\, t)\, \eta_{00} = \boldsymbol{- \frac{c^4}{16\pi G_{eff}} k_{Uni}^2 \cos(k_{Uni}\, t)} \tag{2.106}$$

- The negative sign describes a global energy density composed of dark energy, which exerts a repulsive force. The universe will continue to expand at an accelerating rate as long as dark energy exists. At the beginning of this period, the amount of dark energy is at its maximum, which maximizes the acceleration. At the point of maximum expansion, however, the acceleration decreases to zero because the amount of dark energy reaches zero.

c) $\boldsymbol{T_{0i}^{(\text{global})} = 0}$ no off-diagonal terms in the global part of the metric (2.107)

<u>Vector term:</u>

$$T_{\mu\nu}^{(\text{Vector})} = \frac{1}{4\pi}\left(F_{\mu}^{\lambda a}\, F_{\nu a\lambda} - \frac{1}{4}\, g_{\mu\nu}^{(4)}\, F_{\rho\sigma}^{a}\, F^{a\rho\sigma}\right)$$

a) $$T_{00}^{(\text{Vector})} = \frac{1}{4\pi}\left(F_{0}^{\lambda a}\, F_{0a\lambda} - \frac{1}{4}\, g_{00}^{(4)}\, F_{\rho\sigma}^{a}\, F^{a\rho\sigma}\right) = \frac{1}{4\pi}\left(g^{\lambda\mu}\, F_{0\mu}^{a}\, F_{0\lambda a} - \frac{1}{4}\, g_{00}^{(4)}\, F_{\rho\sigma}^{a}\, F^{a\rho\sigma}\right)$$

Lagrange solution with Si:

$\mathcal{L}_{\text{Vector}} = -\frac{1}{4}\frac{1}{\mu_0}\, F_{\mu\nu}^{a}\, F^{a\mu\nu}$ with the convention: $\frac{E}{c}$, if $g_{00}^{(4)} = (-1, 1, 1, 1)$ remains

$$T_{00}^{(\text{Vector})} = \frac{1}{4\pi}\,\frac{1}{\mu_0}\left(\frac{1}{c^2}\, E^2 - \frac{1}{4}\,(-1)\; 2\left(B^2 - \frac{1}{c^2}\, E^2\right)\right)$$

$T_{00}^{(\text{Vector})} = \frac{1}{4\pi}\left(\epsilon_0\, E^2 + \frac{1}{2}\left(\frac{1}{\mu_0}\, B^2 - \epsilon_0\, E^2\right)\right)$ with: $\frac{1}{c^2} = \mu_0\, \epsilon_0$

$\boldsymbol{T_{00}^{(\text{Vector})} = \frac{1}{8\pi}\left(\epsilon_0\, E_a^2 + \frac{1}{\mu_0}\, B_a^2\right)}$ = electromagnetic power density (2.108)

- The tensor behaves like radiation with $p = \frac{\rho}{3}$, similar to a relativistic radiation fluid; its field mediates strong and weak interactions.
- $\frac{1}{2}\,\epsilon_0\, E^2$ or $\frac{1}{2}\,\frac{1}{\mu_0}\, B^2$ – electrical and magnetic energy density
- $\frac{1}{4\pi}$ – by convention, in classical SI electrodynamics, this becomes a factor 1
- Index a – from A_{μ}^{a}; a originates from the compact wave-field dimensions and, following compactification, denotes the gauge group index

c) $\boldsymbol{T_{0i}^{(\text{Vector})} = \frac{1}{4\pi}\,\epsilon_0\,(E \times B)_i}$ (2.109)

<u>Derivation of the *ρ*-scalar and *p*-scalar from the FSM geometry:</u>

The scalar tensor $T_{\mu\nu}^{(\text{scalar})}$ describes the gravitational source arising from relativistic fields, including kinetic energy, potential energy, the mass term, and dark, uncoupled energy. The *ρ*-scalar and *p*-scalar are key quantities for the cosmological interpretation of the scalar contribution.

- *ρ*-scalar – energy density from the contribution $\rho = T_{00}$ (time-time component); it measures how much energy is stored per unit volume in the individual scalar fields and determines the gravitational force exerted by the scalar fields, including all three generations and uncoupled energy
- *p*-scalar – pressure from the term $\rho = T_{ii}$ for $i = 1, 2, 3$ (spatial components, normalized by the metric); it measures the isotropic stresses – for dark energy, $p = -\rho$; negative pressure drives the expansion by determining the acceleration of the expansion.

To derive the two scalars, the FLRW metric is applied. This ensures a homogeneous universe with $g_{\mu\nu}^{(4)}$ = diag(-1, $a^2(t)$, $a^2(t)$, $a^2(t)$) and neglects spatial gradients ($\nabla\phi_n \approx 0$, only $\partial_t \phi_n \neq 0$ for cosmological fields).

a) Derivation of the *ρ* scalar (energy density, T_{00}):

$$T_{00}^{(\text{scalar})} = c\hbar \sum_{n=1}^{3} [\partial_0 \phi_n \partial_0 \phi_n - \frac{1}{2} g_{00}^{(4)} (\partial^0 \phi_n \partial_0 \phi_n + 2V(\phi_n)_m + 2V(\phi_n)_{pot} + 2V(\phi_n)_{dark})]$$

- $\partial_0 = \partial t$ (Time derivation) → $g_{00}^{(4)} = -1$ with: $c = 1$
- $\partial_0 \phi_n \partial_0 \phi_n = (\partial_t \phi_n)^2$
- $\partial^0 \phi_n \partial_0 \phi_n = -(\partial_t \phi_n)^2$
- $-\frac{1}{2} g_{00}^{(4)} = -\frac{1}{2} \cdot (-1) = \frac{1}{2}$

Insert:

$$T_{00}^{(\text{scalar})} = c\hbar \sum_{n=1}^{3} [(\partial_t \phi_n)^2 + \frac{1}{2} ((-\partial_t \phi_n)^2 + 2V(\phi_n)_m + 2V(\phi_n)_{Pot} + 2V(\phi_n)_{dark})]$$

Complete with *c*-renormalization (kinetic + potential energy):

$$\mathbf{T_{00}^{(scalar)} = c\hbar \sum_{n=1}^{3} c^2 [\frac{1}{2} (\partial_t \phi_n)^2 + V(\phi_n)_m + V(\phi_n)_{pot} + V(\phi_n)_{dark}]} \qquad (2.110)$$

with: $\rho_{eff} = \frac{T_{00}^{(\text{scalar})}}{c^2}$

$$\rho_{eff} = c\hbar \sum_{n=1}^{3} [\frac{1}{2} (\partial_t \phi_n)^2 + V(\phi_n)_m + V(\phi_n)_{pot} + V(\phi_n)_{dark}] \qquad (2.111)$$

c) Derivation of the ρ-scalar (momentum density, T_{0i}):

$$T_{0i}^{(\text{scalar})} = c\hbar\,[\partial_0\,\phi_n\,\partial_i\,\phi_n - \frac{1}{2}\,g_{0i}^{(4)}\,(\dots)] \approx c\hbar\,\partial_0\,\phi_n\,\partial_i\,\phi_n$$

- $\partial_0\,\phi_n = \frac{1}{c}\,\partial_t\,\phi_n$

Complete with c-renormalization (pure gradient flow):

$$\boldsymbol{T_{0i}^{(\text{scalar})}} = c\hbar\,\frac{1}{c}\,\partial_t\,\phi_n\,\partial_i\,\phi_n \boldsymbol{= \hbar\,\partial_t\,\phi_n\,\partial_i\,\phi_n} \tag{2.112}$$

d) Derivation of the p-scalar (pressure), $T_{ii} = p\,a^2(t)$ for i = 1, 2, 3):

$$T_{ii}^{(\text{scalar})} = c\hbar\,\textstyle\sum_{n=1}^{3}\,[0 - \frac{1}{2}\,g_{ii}^{(4)}\,(\partial^\rho\,\phi_n\,\partial_\rho\,\phi_n + 2V(\phi_n)_m + 2V(\phi_n)_{pot} + 2V(\phi_n)_{dark})]$$

- $\partial_i\,\phi_n = 0$ (neglected, homogeneous)
- $g_{ii}^{(4)} = a^2(t)$ (positive)
- $\partial^\rho\,\phi_n\,\partial_\rho\,\phi_n = (\partial_t\,\phi_n)^2$

Insert:

$$T_{ii}^{(\text{scalar})} = -\frac{1}{2}\,c\hbar\,a^2(t)\,\textstyle\sum_{n=1}^{3}\,[(-\,\partial_t\,\phi_n)^2 + 2V(\phi_n)_m + 2V(\phi_n)_{pot} + 2V(\phi_n)_{dark})]$$

$$T_{ii}^{(\text{scalar})} = c\hbar\,a^2(t)\,\textstyle\sum_{n=1}^{3}\,[\frac{1}{2}\,(\partial_t\,\phi_n)^2 - V(\phi_n)_m - V(\phi_n)_{pot} - V(\phi_n)_{dark})] \tag{2.113}$$

$$p = \frac{T_{ii}^{(\text{scalar})}}{a^2(t)}$$

$$p = c\hbar\,\textstyle\sum_{n=1}^{3}\,[\frac{1}{2}\,(\partial_t\,\phi_n)^2 - V(\phi_n)_m - V(\phi_n)_{pot} - V(\phi_n)_{dark})] \tag{2.114}$$

a) Total energy density ρ_{eff}:

$$\rho_{eff} = \frac{T_{00}^{eff}}{c^2}$$

$$\rho_{eff} = \rho - \frac{p}{c^2} - \frac{c^2}{16\pi G_{eff}}\,k_{Uni}^2\,\cos(k_{Uni}\,t) + \frac{1}{8\pi}\,\frac{1}{c^2}\,(\epsilon_0\,E_a^2 + \frac{1}{\mu_0}\,B_a^2) + c\hbar\,\textstyle\sum_{n=1}^{3}\,[(\frac{1}{2}\,(\partial_t\,\phi_n)^2 + V(\phi_n)_m + V(\phi_n)_{pot} + V(\phi_n)_{dark})] \tag{2.115}$$

Relativistic:

$$\rho_{eff} = \frac{1}{c^2} [(\rho\ c^2 + p) \frac{1}{\sin^2(kt)} - p] - \frac{c^2}{16\pi G_{eff}} k_{Uni}^2 \cos(k_{Uni}\ t) + \frac{1}{8\pi} \frac{1}{c^2} (\epsilon_0\ E_a^2 + \frac{1}{\mu_0}\ B_a^2) + \frac{1}{\sin^2(kt)}\ c\hbar \sum_{n=1}^{3} [(\frac{1}{2} (\partial_t\ \phi_n)^2 + V(\phi_n)_m + V(\phi_n)_{Pot} + V(\phi_n)_{dark})] \quad (2.116)$$

- Unit: $\frac{kg}{m\ s^2}$

c) Total pulse density g_{eff}:

$$g_{eff} = \frac{T_{0i}^{eff}}{c}$$

$$g_{eff} = (\rho + \frac{p}{c^2})\ V_i \frac{1}{c} + \frac{1}{4\pi} \frac{1}{c}\ \epsilon_0\ (E \times B)_i + \hbar \sum_{n=1}^{3} \frac{1}{c}\ \partial_t\ \phi_n\ \partial_i\ \phi_n \quad (2.117)$$

Relativistic: (2.118)

$$g_{eff} = \frac{1}{c^2 \sin^2(kt)} [(\rho\ c^2 + p)\ V_i \frac{1}{c}] + \frac{1}{4\pi} \frac{1}{c}\ \epsilon_0\ (E \times B)_i + \frac{1}{\sin(kt)} [\hbar \sum_{n=1}^{3} \frac{1}{c}\ \partial_t\ \phi_n\ \partial_i\ \phi_n]$$

- Unit: $\frac{kg}{m^2\ s}$

Addendum on Compactification:

All observable 4-dimensional phenomena, such as gravity, electromagnetism, particle masses, and other forces, arise as a low-energy approximation of 7-dimensional Field-Space-Mechanics following integration over the compact dimensions. In this process, gauge potentials of the 7-dimensional metric reduce to effective gauge fields in the 4-dimensional space-time. Electromagnetism and the weak force do not arise as independent fundamental fields, but as geometric effects of the compact dimensions, just as different Fourier modes generate the respective coupling structure.

The Fourier modes of the compact coordinates give rise to discrete particle masses, which manifest as heavy states. Dark energy appears as a natural oscillation of the compact dimensions. Photons and other particles fundamentally exist as waves in the compact wave-field ($F_{4\text{-}6}$), but appear as discrete objects in the visible 4-dimensional particle-field ($F_{1\text{-}3}$ plus time). This provides a geometric explanation of the wave-particle duality. Gravity itself acts primarily in the visible 4D realm and is preserved as classical curvature.

By applying the modes in the FSM model, it becomes possible to make new predictions about heavy particles that may be relevant to fusion processes for energy production or to future propulsion technologies.

FSM does not quantize gravity. Instead, gravity is retained as classical geometric curvature, while all quantum effects and the unification of interactions are explained by the dynamic geometry of the compact wave-field dimensions—rotation, coupling, and modes. The model thus provides a geometric bridge between classical gravity and the quantum world.

11. Development of the wave equation for gravitational waves

<u>The starting point is the field equation of the FSM:</u>

$$R_{MN} - \frac{1}{2} g_{MN} R = \frac{8\pi G}{c^4} T_{MN} \qquad (2.78)$$

<u>Linear approximation around the flat metric (perturbation):</u>

Gravitational waves are small disturbances, so the Minkowski metric η_{MN} is linearized.

$g_{MN} = \eta_{MN} + h_{MN}$ with: $h_{MN} << 1$

- η_{MN} – 7D symmetric Minkowski metric; η_{MN} = diag(-1, 1, 1, 1, 1, 1, 1);
  index: M, N = 0,…, 6
- h_{MN} – metric disturbance; describes gravitational waves as a small deviation from the flat metric; neglect of higher-order terms such as h^2, since waves are weak → Advantage: Simplification of the calculation to a wave equation

<u>Christoffel symbols in linear approximation:</u>

- $\Gamma^{\lambda}_{NM} = \frac{1}{2} h^{\lambda P} (\partial_M h_{NP} + \partial_N h_{MP} - \partial_P h_{MN})$ (2.119)
- Γ^{λ}_{NM} – Christoffel symbol (non-tensorial); unit: $\frac{1}{\text{m}}$; describes how vectors change during parallel transport in curved space-time; index above λ = 0,…, 6; below M, N = 0,…, 6
- $h^{\lambda P}$ – inverse Minkowski metric; dimensionless; raises indices λ, P = 0,…, 6; $h^{\lambda P}$ = diag(-1, 1, 1, 1, 1, 1, 1); sum over P is implicitly Einstein convention
- Terme $\partial_M h_{NP}$ – partial derivatives according to x^M; unit: $\frac{1}{\text{m}}$; measures the change in disturbance in the direction of M
- Approximation: no ΓΓ terms, since linear; simplified wave equation

<u>Ricci tensor in linear approximation:</u>

Ricci tensor:

$$R_{MN} \approx \partial_P \Gamma^P_{MN} - \partial_N \Gamma^P_{MP}$$

After the onset of Christoffel signs and contractions:

$$R_{MN} = \frac{1}{2} (\partial_P \partial_M h^P_N + \partial_P \partial_N h^P_M - \partial_P \partial^P h_{MN} - \partial_M \partial_N h^P_P) + R^{(\text{global})}_{MN} \qquad (2.120)$$

$$R^{(\text{global})}_{MN} = \frac{1}{2} \eta^{\lambda P} (\partial_M [\cos(k_{Uni}\, t)]\, \eta_{NP} + \partial_N [\cos(k_{Uni}\, t)]\, \eta_{MP} - \partial_P [\cos(k_{Uni}\, t)]\, \eta_{MN})$$

- R_{MN} – Ricci tensor in linear approximation; unit: $\frac{1}{\text{m}^2}$; the tensor is the source of the wave equation
- $\partial_P \partial_M h^P_N$ or $\partial_P \partial_N h^P_M$ – displacement terms; describe how the disturbance in the P direction affects the curvature between M and N; unit: $\frac{1}{\text{m}^2}$
- $- \partial_P \partial^P h_{MN}$ – D'Alembert operator $\square\, h_{MN} = - \partial_P \partial^P h_{MN} = - \partial t^2 h_{MN} + \nabla^2 h_{MN}$; unit: $\frac{1}{\text{m}^2}$; wave term; index $P = 0,\ldots, 6$
- $- \partial_M \partial_N h^P_P$ – This term corrects the trace components (scalar components) of the disturbance $h = h^P_P$, which is dimensionless, and takes scalar modes into account; in the trace-corrected calibration $h = 0$, the term vanishes
- $R^{(\text{global})}_{MN}$ – Global terms describe the cosmic oscillation of the entire space-time. They act as a background curvature that modulates all waves and generates an additional longitudinal component
- Approximation: no ΓΓ terms

<u>Trace-free disturbance for 7D:</u>

Trace-free transverse (TT) calibration:

$h = \eta^{MN} h_{MN} = 0$

Trace-cleaned disturbance:

$$\bar{h}_{MN} = h_{MN} - \frac{1}{2} \eta_{MN}\, h$$

- $\bar{h}_{MN}$ – trace-free disturbance; unit: $\frac{1}{\text{m}^2}$

The following now holds for the Ricci tensor and the Ricci scalar:

R_{MN}: Translation terms disappear

$R \approx 0$

<u>Substitution into the FSM field equation:</u>

$$R_{MN} - \frac{1}{2} g_{MN}\, R = \frac{8\pi G}{c^4} T_{MN}$$

In a vacuum: $T_{MN} = 0$. After substitution, the following remains:

$$- \frac{1}{2} \partial_P \partial^P \bar{h}_{MN} + R_{MN}^{(global)} = 0$$

$$\Box_{(7)} \bar{h}_{MN} = 2 R_{MN}^{(global)} \quad (2.121)$$

<u>The D'Alembert operator in 7D in general:</u>

$$\Box_{(7)} = \eta^{MN} \partial^M \partial_M = - \frac{1}{c^2} \partial_t^2 + \nabla^2_{(3)} + \nabla^2_{(3,\ compact)} = - \frac{1}{c^2} \partial_t^2 + \nabla^2_{(3)} + \partial_a \partial^a$$

- $\Box_{(7)} = \partial^M \partial_M = - \partial_t^2 + \nabla^2 + \partial_a \partial^a$ – the D'Alembert operator in 7D; unit: $\frac{1}{m^2}$; describes wave propagation
- Running Index a = 4, 5, 6 regarding the compact wave-field dimensions

<u>Gauge selection - harmonic gauge in a vacuum ($T = 0$) in FSM:</u>

$$\partial^M \bar{h}_{MN} = 0 \quad (2.122)$$

- $\partial^M \bar{h}_{MN}$ – diversity; unity: $\frac{1}{m}$; Gauge condition; eliminates redundant degrees of freedom; summation index $M = 0,\dots, 6$

In a vacuum ($T = 0$), this gauge is related to the field equation by:

$$\Box_{(7)} \bar{h}_{MN} = 2 R_{MN}^{(global)} = - k_{Uni}^2 \cos(k_{Uni}\, t)\, \eta_{MN} \quad (2.123)$$

- $\Box_{(7)} \bar{h}_{MN}$ – trace-free metric solution in compact wave-field dimensions

  $\Box_{(7)} \bar{h}_{MN} = 2 R_{MN}^{(global)}$

  $\Box_{(7)} \bar{h}_{MN} = 0$ for an GTR solution that neglects the global component according to the ‘Mach principle’

<u>Polarization and degrees of freedom in the FSM:</u>

In TT-Gauge (transvers-tracelos, extended to 7D, but reduced to 4D), the result is:

- In 4D-GTR:
  - $h_{\mu\nu}$ has two degrees of freedom (+ and × polarization for waves in the z-direction)
    $h_{xx} = - h_{yy} = h_+ \cos(wz – kt)$
    $h_{xy} = h_{yx} = h_x \cos(wz – kt)$
  - h_+, h_x – amplitudes; dimensionless; both independent polarizations of gravitational waves (spin-2) arise
  - w – wave number; unit: $\frac{1}{m}$; number of wavelengths per meter; $w = 2\pi/\lambda$; with: λ as wavelength

- z – location coordinate in propagation direction; unit: m
- k – circular frequency; unit: $\frac{\text{rad}}{\text{s}}$; angular velocity of oscillation
- t – time; unit: s
- ($wz – kt$) – phase of the wave; wz – spatial; kt – temporal; difference means that the wave travels in the positive z-direction

- In 7D-FSM:
  - h_{MN} – The global Ricci tensor acts as a cosmic "pressure wave" that exerts uniform pressure in all directions. This isotropic excitation cannot generate pure tensor waves (which are trace-free), but it can generate vector fields and scalar fields.
  - h_{MN} is dimensionless
  - From the inhomogeneous $\square_{(7)}\bar{h}_{MN}$, decomposing the perturbation into irreducible representations yields a structure with 14 degrees of freedom, with gauge degrees of freedom already subtracted:
    tensor h_{ij} (2), vector A_μ^a (6+3), scalar ϕ_n (3)
  - Indices: i, j = 1, 2, 3; a = 4, 5, 6; μ = 0, 1, 2, 3; n = 1, 2, 3
  - **Tensor polarization $h_{\mu\nu}$** has two degrees of freedom, as in GTR, and produces **two massless transverse** waves with spin-2:
    + polarization ($h_{xx} = - h_{yy}$)
    x polarization ($h_{xy} = h_{yx}$)
  - **Vector polarizations** arise from the off-diagonal metric components $h_{\mu a} = A_\mu^a$
    These correspond to massive vector fields originating from the compact wave-field dimensions. Nine degrees of freedom: 3 × 4 = 12, minus 3 Gauge at μ = 0) provide:
    Six transverse waves with photon-like modes, but massive with spin-1
    Three longitudinal waves are massive modes, helicity 0
  - **Scalar polarizations** from the modes of the compact wave-field dimensions result in three degrees of freedom with ϕ_1, ϕ_2, ϕ_3. Scalar fields possess "hidden" longitudinal waves with ϕ_n (spin = 0, generates volume change). This is not Gauge, but physically results from the geometric deviation from the dimensional plane D_{56} by stabilizing the three vectorial longitudinal waves. **Stabilization mechanism** of the FSM.
  - **Three longitudinal waves** scalar
    In summary:
    2 massless transverse tensor waves (classical GTR)
    6 massive transverse vector waves
    3 massive longitudinal vector waves
    3 massive scalar waves that couple with the longitudinal vectors
  - **Massive dispersion** of the transverse wave with angular frequency:

$k^2 = w^2 c^2 + 4\pi^2 m^2 \frac{c^4}{h^2}$; unit: $\frac{1}{s}$; w – wave number, unit: $\frac{1}{m}$

a) For high frequencies $w >> m$: $k \approx c\, w$ → dispersion-free; $v \approx c$

b) For low frequencies $w << m$: $k \approx 2\pi\, m \frac{c^2}{h}$; wave propagates slowly (v < c); dispersion is strong

Consequently:

1) Transverse wave packets disperse because different frequencies have different speeds:

$v = c^2 \frac{w}{k} < c$

2) Object nearby → high gravitational interaction; object in the distance → weak or no attraction

Wave equation for degrees of freedom and dispersion assignment:

The compact metrics component

$(1 + \cos(k_{Uni}\, t))$

acts as a time-dependent amplitude modulation on the entire wave, regardless of whether it is transverse or longitudinal.

a) 2 massless transverse tensor waves (from $h_{\mu\nu}$):

Wave equation:

$\Box\, h_{ij}^{TT} = 0$

Whereby:

$(\partial_t^2 - c^2\, \nabla^2_{(3)})\, h_{ij}^{TT} = 0$

Dispersion:

$k^2 = w^2 c^2$ (massless, $v = c$) (2.124)

Functional shape with wave in the z-direction:

$h_{ij}(t, z) = [1 + \cos(k_{Uni}\, t)]\, [h_+ e^{i(wz-kt)}\, e_{ij}^{+} + h_x\, e^{i(wz-kt)}\, e_{ij}^{x}]$

In its real form:

$h_{ij}(t, z) = [1 + \cos(k_{Uni}\, t)]\, [h_+ \cos(w_z z - kt)\, e_{ij}^{+} + h_x \cos(w_z z - kt)\, e_{ij}^{x}]$ (2.125)

With transverse trace-free polarization tensors:

$e_{xx}^{+} = - e_{yy}^{+} = 1$; $e_{xy}^{x} = e_{yx}^{x} = 1$

- Indices: i, j = 1, 2, 3 (for x, y, z)
- $e_{ij}^{+} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$; $e_{ij}^{\times} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
- No phase shift, because these modes do not couple directly to the internal scalar fields ϕ_n or to the phases of the compactification. These polarizations “recognize” the oscillation $(1 + \cos(k_{Uni}\ t))$ only as a global, isotropic modulation of the metric amplitude.
- $(1 + \cos(k_{Uni}\ t))$ – modulation fluctuation; pulsating, uniform in space-time relative to nominal time t; defines the temporal sequence for spatial deformation
- h_+ and $h_\times$ – oscillate and propagate rapidly; these carry the wave information for the angular frequency k and wave number w
- e_{ij}^{+} and $e_{ij}^{\times}$ – polarization matrices; define spatial deformation direction

b) 6 solid transverse vector waves (from A_μ^a; a = 4, 5, 6; μ = 0, 1, 2, 3):

These arise from the off-diagonal components $h_{\mu a}$. For each direction a, there are two transverse polarization states for μ.

Wave equation (Proca-like for massive vector fields in 4D):

$$\Box_{(4)} A_\mu^a - \partial_\mu (\partial^\nu A_\nu^a) + 4\pi^2 m_a^2 \frac{c^2}{h^2} A_\mu^a = 0 \quad \text{with: } m = \frac{n\, h}{2\pi\, c\, R} \quad \text{after compactification}$$

With Lorentz condition:

$$\partial^\mu A_\mu^a = 0$$

Dispersion:

$$k^2 = w^2 c^2 + 4\pi^2 m^2 \frac{c^4}{h^2}; \text{ (massive, } v < c) \tag{2.126}$$

Functional form results in a flat wave in the z-direction as transverse modes:

$$A_i^a(t, z) = A_0^a \,[1 + \cos(k_{Uni}\, t)]\, e^{i(wz-kt)} \sum_{pol=1,2} \epsilon_i^{pol}$$

In its real form:

$$A_i^a(t, z) = A_0^a \,[1 + \cos(k_{Uni}\, t)] \cos(w_z z - kt) \sum_{pol=1,2} \epsilon_i^{pol} \tag{2.127}$$

- Indices: $i = x, y$ (transverse); unit vector: $\epsilon_x^{1,0,0} = (1, 0, 0)$, $\epsilon_y^{0,1,0} = (0, 1, 0)$, are the polarization vectors that oscillate orthogonally to w; for three fields: $3 \times 2 = 6$ modes
- Gauge boson-like forces

c) 3 solid longitudinal vector waves (from A_μ^a; a = 4, 5, 6; μ = 0, 1, 2, 3):

Wave equation, Proca-like as in b), but with longitudinal component:

$$\square_{(4)} A_\mu^a - \partial_\mu (\partial^\nu A_\nu^a) + 4\pi^2 m_a^2 \frac{c^2}{h^2} A_Z^a = 0$$

With: z = direction of propagation; it couples to the other components via the Lorentz condition

Dispersion:

$$k^2 = w^2 c^2 + 4\pi^2 m^2 \frac{c^4}{h^2}; \text{ (massive, } v < c) \quad (2.128)$$

Functional shape results in a flat wave in the z-direction:

$$A_z^a(t, z) = A_0^a [1 + \cos(k_{Uni}\, t)]\, e^{i(wz-kt)}\, \epsilon_z$$

In its real form:

$$A_z^a(t, z) = A_0^a [1 + \cos(k_{Uni}\, t)] \cos(w_z z - kt)\, \epsilon_z \quad (2.129)$$

- Parallel to w; 1 per field; 3 in total because of a = 4, 5, 6
- Stabilization of compactification through coupling to scalars
- Unit vector: $\epsilon_z^{0,\,0,\,1}$ = (0, 0, 1)

d) 3 massive scalar waves (from ϕ_1, ϕ_2, ϕ_3):

Wave equation (Klein-Gordon equation for massive scalars):

$$\square_{(4)} \phi_n + m_n^2 \phi_n = 0$$

Dispersion:

$$k^2 = w^2 c^2 + 4\pi^2 m_n^2 \frac{c^4}{h^2}; \text{ (massive, } v < c) \quad (2.130)$$

Functional shape results in a flat wave:

$$\phi_n(t, z) = \phi_0 [1 + \cos(k_{Uni}\, t)]\, e^{i(wz-kt)}$$

In its real form:

$$\phi_n(t, z) = \phi_0 [1 + \cos(k_{Uni}\, t)] \cos(w_z z - k_n t) \quad (2.131)$$

- Scalar, isotropic, and longitudinal; n per field; n = 1, 2, 3; coupling to longitudinal vectors for stabilization

Result of the wave equation in FSM as in GTR:

$\bar{h}_{\mu\nu} = 0$; this means for the gravitational wave:

- Propagation at maximum speed $V_{max} = c$
- Transverse and trace-free; only spatial disturbances, no volume change
- Two polarizations have spin-2 field (graviton), acts like a quadrupole
- The mass of an object is modeled in the wave-field. For metric reasons, an **object in the particle-field** is merely a **massless projection**.

Advantages of FSM terms:

- **cos($k_{Uni}\, t$)** – generates dynamic vacuum energy, allowing this term to modulate gravitational waves (oscillating amplitude), explains invisible (including dark) energy waves or variations in propagation, and thus explains possible deviations of GTR waves (e.g., dispersion due to compact dimensions)
- **Scalar terms $V(\phi_n)$:** generate additional scalar waves (spin-0) that couple with gravitational waves. Advantage: FSM may be able to explain multi-messenger signals (e.g., scalar waves that modulate dark energy) and the Higgs mass without fine-tuning (from R – radius of the compact wave-field dimensions).
- **Wave components that arise additionally from the FSM metric carry the inertial and interaction forces, which are ultimately registered in the particle-field. In the particle-field, these act as a projection of their restoring forces in the wave-field. Restoring forces are self-interactions arising from a disturbance caused in the particle-field or in the wave-field. A geometric displacement from their rest position in space-time causes such a disturbance.**

12. The field radius *r*

The field radius r is the spatial scale of action of a relativistic field, which arises from the curvature of space-time.

The starting point is the FSM field equations:

$$R_{MN} - \frac{1}{2}\, g_{MN}\, R = \frac{8\pi G}{c^4}\, T_{MN}$$

Transition to perturbative approximation in weak fields:

$$\partial_\lambda \Gamma^\lambda_{NM} - \partial_N \Gamma^\lambda_{\lambda M} - \frac{1}{2}\, g^0_{MN}\, g^{PQ}_0\, (\partial_\lambda \Gamma^\lambda_{QP} - \partial_Q \Gamma^\lambda_{\lambda P}) = \frac{8\pi G}{c^4}\, T_{MN}$$

<u>Metric approach for static mass with rotation in FSM:</u>

For a point-like mass *M*, the metric perturbation is

$$h_{00} = 2\,\frac{G\,M}{c^2\,r}\ , \tag{2.132}$$

while the metric disturbance in the case of rotation is:

$$h_{\mu\nu} = \frac{G\,M}{c^2\,r}\,(1 + \cos(kt + \beta))\,\delta_{\mu i}\,\delta_{\nu j} \text{ (aus Punkt 1.)}$$

The mean value over the rotation is <1 + cos(kt)> = 1 for $\beta = 0$.

- $h_{\mu\nu}$ – metric perturbation causes weak gravity; μ, ν = 0, 1, 2, 3
- $\frac{G\,M}{c^2\,r}$ – gravitational potential; causes $\frac{1}{r}$ – dependence
- 1+ cos(kt) – modulation causes a dynamic halving; for: $\beta = 0$
- $\delta_{\mu i}\,\delta_{\nu j}$ – Kronecker; specifies the direction of the spatial components; i, j = 1,2,3

<u>Insert into the linearized equation for the field radius *r* :</u>

In a vacuum with $T_{MN} \approx 0$ solves the equation for h_{00} using a Newton approximation:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\nabla^2\,h_{00} = 4\pi\,G_\rho$$

$$\nabla^2\,h_{00} = \frac{8\pi G}{c^4}\,\rho \qquad \text{with: } \rho = M\,\delta^3(r)$$

For a point mass, see also point 1:

$$h_{00} = 2\,\frac{G\,M}{c^2\,r} \qquad \text{with the rotation fixed at the point of maximum: } 1 + \cos(kt = 0) = 2$$

In the case of halving by means of rotation in the wave-field:

$$h_{00} = \frac{G\,M}{c^2\,r} \qquad \text{with: } 1 + \cos(kt) = 1 \tag{2.133}$$

- $\nabla^2\,h_{00}$ – Laplace of the metric disturbance; unit: $\frac{1}{m^2}$; potential equation
- ρ – density; unit: $\frac{kg}{m^3}$; for the mass source
- $\delta^3(r)$ – delta function; unit: $\frac{1}{m^3}$; for the point source
- $2\,\frac{G\,M}{c^2\,r}$ – classical solution for the disturbance according Schwarzschild

- $\frac{1}{2}$ – for halving with the average rotation factor

Result for the field radius r :

$$r = \frac{GM}{c^2} \quad (2.134)$$

Implications for black holes:

The effective event horizon for rotating objects is reached at $r = \frac{GM}{c^2}$, not at $r_S = \frac{2GM}{c^2}$. This also explains why Kerr-like rotating solutions appear more natural in the FSM. The "half" Schwarzschild radius r_S is a direct consequence of the dynamic rotation of the compact dimensions.

While in classical GTR the field radius $r \rightarrow$ leads to a singularity, the FSM model, with its compact wave-field structure, prevents the dimensions y_4, y_5, y_6 from collapsing. Scalar fields ϕ_n and the oscillation of the compact volume determinant generate a stabilization potential $V(\phi_n)$, which builds up a strong restoring potential at very small radii. The geometric parameter with compact volume radius R never becomes zero, but oscillates periodically. Information is preserved in the compact wave-field dimensions and is never "lost." Consequently, in FSM there is no singularity at the center of a black hole; instead, it is a highly complex, oscillating wave-field structure.

The local factor $(1 + \cos(kt + \beta))$ generates an oscillating disturbance around the black hole. Consequently, the event horizon "pulsates" by periodically expanding and contracting. This leads to a periodic change in the effective shadow size and the Hawking temperature.

Hawking radiation arises from the oscillation of the horizon metric and its coupling to the scalar fields ϕ_n. It is therefore not purely thermal, but contains an oscillating component (modulated by $(1+ \cos(kt))$).

The global factor $(1 + \cos(k_{Uni}\, t))$ additionally modulates the entire metric periodically. Upon reaching the maximum expansion of the universe with: $(1+ \cos(k_{Uni}\, t = 90°))$, the configuration relative to the dimensional plane D_{56} changes such that no potential can be generated parallel to the fourth dimension. Consequently, structures without gravitational or electric potential dissolve. Black holes dissolve upon reaching the maximum spatial expansion. Information contained within them is subsequently released again.

Taking into account the additional wave properties in the FSM described in point 11, future detectors (LISA, Einstein Telescope) could reveal additional signals or damping modes.

13. The angular frequency *k*

The circular frequency k is the invariant clock frequency of the sinusoidal field rotations that drive the dynamics of curvature, particles, and expansion. The circular frequency k is thus a characteristic frequency at which a mass M causes its own space-time to “oscillate” when viewed as an isolated system.

<u>The starting point is the FSM field equations:</u>

$$R_{MN} - \frac{1}{2} g_{MN} R = \frac{8\pi G}{c^4} T_{MN}$$

<u>Transition to perturbative approximation for weak fields around an object:</u>

$$g_{MN} = [1 + \cos(k_{Uni}\, t)]\, \eta_{MN} + h_{MN} \qquad h_{MN} << 1$$

The local perturbation around the object, in a linear approximation, is:

$$h_{\mu\nu} = \frac{G\,M}{c^2\, r} (1 + \cos(kt + \beta))\, \delta_{\mu\nu} \qquad h_{00} = \frac{G\,M}{c^2\, r} \qquad \text{with: } 1 + \cos(kt) = 1$$

Gravitational potential:

$$g_{00} \approx - c^2 \left(1 - \frac{G\,M}{c^2\, r}\right)$$

$$\mathbf{V_{grav}(r) = - \frac{G\,M}{r}}$$

Oscillation potential arising from the compact wave-field dimensions with:

$$E_{kin} = \frac{1}{2}\, m\, v^2 = \frac{1}{2}\, m\, (kr)^2$$

$$\mathbf{V_{rot}\,(r) = \frac{1}{2}\,(kr)^2}$$

<u>Effective potential $V(r)_{eff}$:</u>

$$V(r)_{eff} = V_{grav}(r) + V_{rot}(r)$$

$$V(r)_{eff} = - \frac{G\,M}{r} + \frac{1}{2}\,(kr)^2$$

<u>Condition for the minimum:</u>

$$\frac{dV(r)}{dr} = 0 = - \frac{G\,M}{r^2} + k^2\, r$$

$$\boxed{k = \sqrt{\frac{G\,M}{r^3}} = \frac{c^3}{G\,M}} \qquad (2.135)$$

Space-time constant

$k_{Uni} \sim \frac{1}{r_{Uni}}$ A closed spherical universe requires precisely aligned universal constants so that a circular frequency k_{Uni} is exactly inversely proportional to the maximum volume radius R_{Uni} of the universe.

$$r_{Uni}\, k_{Uni} = \frac{G\, M_{Uni}}{c^2} \sqrt{\frac{G\, M_{Uni}}{r_{Uni}^3}} = \sqrt{\frac{(G\, M_{Uni})^3\, (c^2)^3}{(G\, M_{Uni})^3\, (c^2)^2}} = c$$

$$r_{Uni}\, k_{Uni} = r_{obj}\, k_{obj} = \text{constant} = c = 299792458\, \frac{\text{m}}{\text{s}} \qquad (2.136)$$

The **space-time constant** is the product of the field radius r and the angular frequency k of an object.

14. Photon Subspace Theory

Photons are part of the universe's 7-dimensional photon field, which unfolds within the particle-field $F_{1\text{-}3}$ and the wave-field $F_{4\text{-}6}$. Mathematically, multiple 4-dimensional subspaces can exist within a 6-dimensional hollow sphere structure. The photon subspace theory describes photons within the geometry of the invisible wave-field $F_{4\text{-}6}$, where they oscillate as null geodesic waves and resolve the wave-particle duality through sinusoidal rotations.

4-dimensional subspaces in 7D:

The 4-dimensional subspaces each occupy two spatial dimensions in the compact component of the momentum-energy tensor T for the mathematical rotation in the wave-field and the particle-field. For the coupling component, in the visible case—which has a point of contact with the dimensional plane D_{56} —three dimensions correspond to the particle-field and only one dimension to the wave-field. Hidden matter, on the other hand, has only one dimension in the visible particle-field, while three dimensions correspond to the wave-field.

Wave equations for gravity from Point 11:

Transverse wave remains with: $h_{\mu\nu} = 0$ Mass = 0

Transverse vector wave: $\square_{(4)}\, A^a_\mu - \partial_\mu\, (\partial^\nu\, A^a_\nu) + 4\pi^2\, m^2_a\, \frac{c^2}{h^2}\, A^a_\mu = 0$ massiv

Longitudinal vector wave: $\square_{(4)}\, A^a_\mu - \partial_\mu\, (\partial^\nu\, A^a_\nu) + 4\pi^2\, m^2_a\, \frac{c^2}{h^2}\, A^a_Z = 0$ massiv

Longitudinal scalar wave: $\square_{(4)}\,\phi_n + m_n^2\,\phi_n = 0$ Masse abhängig Mode n

Dispersion of the transverse wave with angular frequency: $k^2 = w^2\,c^2 + 4\pi^2\,m_a^2\,\frac{c^4}{h^2}$;

unit: $\frac{1}{s}$; w – wavenumber, unit: $\frac{1}{m}$

<u>Metric in the photon subspace, separation between the wave-field and the particle-field:</u>

$$ds^2 = [1 + \cos(k_{Uni}\,t)]\,[-\,c^2\,dt^2 + g_{ij}\,dx^i\,dx^j + \gamma_{pq}\,dy^p\,dy^q + 2(A_\mu^a + \delta A_\mu^a)\,dy_a\,dx^\mu] \qquad (2.137)$$

$g_{ij} = \eta_{ij} + h_{ij} = \eta_{ij} + \frac{GM}{c^2 r}\,(1 + \cos(kt + \beta))$ → visible metric component for local

$\gamma_{pq} = \eta_{pq}$ → compact metric component for local

- Indices: i, j for 2D visible dimensions i, j = 1, 2, 3; p, q for the compact wave-field component in 2D; p, q = 4, 5, 6; however, depending on the linking scenario, only two dimensions are activated at a time
- ds^2 – invariant line element; unit: m²; measures distances in 4D space-time
- $-\,c^2\,dt^2$ – time term; unit: m²; causes causality; c – maximum speed
- g_{ij} – metric in the visible particle-field; results in a 2D visible portion
- $dx^i\,dx^j$ – displacement in the visible region due to coupling between wave-fields $F_{4\text{-}6}$ and particle-fields $F_{1\text{-}3}$; unit: m²; transverse component
- γ_{pq} – metric in the wave-field; produces a 2D wave component
- $dy^p\,dy^q$ – displacement in the compact region; unit: m²; longitudinal component

<u>Summary of the distribution of dimensions:</u>

- Visible particles: max. 3D in the particle-field; max. 2D in the wave-field $D_{14/24/34}$; effect of the transverse wave
- Invisible particles: max. 3D in the wave-field, max. 2D in the particle-field $D_{16/26/36}$; effect of the longitudinal wave
- Photon-compact component: 2D in the wave-field, 2D in the particle-field $D_{45/46/56}$; effect of the longitudinal wave
  Photon coupling component:
  Interaction in D_{56}: 1D in the wave-field, 3D in the particle-field (visible interaction)
- Not in contact with D_{56}: 3D wave-field, 1D particle-field (“dark matter” or hidden matter)

Correlation of the waveform with the field propagation speeds V_5 and V_4:

$V_4 = V_{long} = c\,\cos(kt)$ $V_5 = V_{trans} = c\,\sin(kt)$ $c^2 = V_5^2 + V_4^2$

- ***V_4*** – field propagation speed; unit: $\frac{\mathrm{m}}{\mathrm{s}}$
  - **Longitudinal disturbance component;** orthogonal to/relative to the dimensional plane D_{56}
  - Generates the **field** (directional component) and represents the disturbance acting on the visible field deformation.
  - Upon contact with D_{56}, V_4 is **minimal**. A photon couples maximally with the particle-field. A periodic **matter pulse** (**Chapter 3.2**) is generated between the wave-field and the particle-field, corresponding to the amplitude of their field exchange. A discrete particle is detected.
- ***V_5*** – field propagation speed; unit: $\frac{\mathrm{m}}{\mathrm{s}}$
  - **Transverse, visible field deformation**; parallel to the dimensional plane D_{56}
  - Measures the **visible intensity** of the photon in the particle-field
  - When D_{56} is touched, V_5 is at its **maximum**. Acts like a **field-body oscillation** in the particle-field
  - **A photon** is always emitted when the transverse field deformation reaches its maximum.
- $\cos(kt)$ or $\sin(kt)$ – The phases cause the transition between visible and invisible matter.

The incorporation of both field propagation speeds into the wave-field resolves the wave-particle duality and explains dark matter as “false” recombination. The photon mass breaks U(1) symmetry. Photons are no longer massless and no longer have infinite range.

Geometrically dependent mass with its amplitude and as a periodic disturbance:

$$m_{pho} = \frac{n\,h}{2\pi\, c\, R_{pho}}$$

$$m_{pho}(t) = \frac{n\,h}{2\pi\, c\, R_{pho}} \cos(kt + \beta) \qquad (2.138)$$

The modes n are taken into account in the frequency calculation of an object and give rise to mass classes (in the form of generations) that exhibit small jumps in excitation frequencies between groups. Furthermore, the complexity of their particle structure increases continuously.

<u>Zero-geodesic photon motion: local and global:</u>

Photons with $f > f_{min}$ follow ds in the subspace of complementary dimensions:

Global d$s \approx 0$:

$$0 \approx - c^2 \mathrm{d}t^2 + g_{ij}\, \mathrm{d}x^i\, \mathrm{d}x^j + \gamma_{pq}\, \mathrm{d}y^p\, \mathrm{d}y^q + 2(A^a_\mu + \delta A^a_\mu)\, \mathrm{d}y_a\, \mathrm{d}x^\mu$$

Local ds = 0:

$$0 = - c^2 dt^2 + g_{ij}\, dx^i\, dx^j + \gamma_{pq}\, dy^p\, dy^q + 2(A^a_\mu + \delta A^a_\mu)\, dy_a\, dx^\mu$$

<u>Field equation in a subspace:</u>

In the weak-field approximation, the Einstein equations are reduced to the wave equations after trace elimination and the adoption of a harmonic gauge:

a) $R_{ij} - \frac{1}{2} g_{ij}\, R = \frac{8\pi G}{c^4}\, T_{ij}$ (2.139)

b) $R_{pq} - \frac{1}{2} \gamma_{pq}\, R = \frac{8\pi G}{c^4}\, T_{pq}$ (2.140)

- T_{ij} ~ transverse tensor (visible)
- T_{pq} ~ longitudinal tensor (compact)
- R_{ij} or R_{pq} – Ricci tensor; unit: $\frac{1}{m^2}$; causes curvature in the respective section
- R – The Ricci scalar is the total trace of the Ricci tensor over the entire metric of the photon subspace
- T_{ij} or T_{pq} – momentum-energy tensor; unit: $\frac{kg}{m\,s^2} = \frac{J}{m^3}$; generates energy from waves

With:

$$R_{MN} = \frac{1}{2} (\partial_P\, \partial_M\, h^P_N + \partial_P\, \partial_N\, h^P_M - \partial_P\, \partial^P\, h_{MN} - \partial_M\, \partial_N\, h^P_P) + R^{(global)}_{MN}$$

$$R^{(global)}_{MN} = \frac{1}{2}\, \eta^{\lambda P}\, (\partial_M [\cos(k_{Uni}\, t)]\, \eta_{NP} + \partial_N [\cos(k_{Uni}\, t)]\, \eta_{MP} - \partial_P [\cos(k_{Uni}\, t)]\, \eta_{MN})$$

$R^{(global)}_{ij} \approx - \frac{1}{2}\, k_{Uni}^2 \cos(k_{Uni}\, t)\, \delta_{ij}$ and $R^{(global)}_{pq} \approx - \frac{1}{2}\, k_{Uni}^2 \cos(k_{Uni}\, t)\, \delta_{pq}$

$R_{ij} \approx \frac{1}{2} (\cancel{\partial_k \partial_i h^k_j} + \cancel{\partial_k \partial_j h^k_i} - \partial_k\, \partial^k\, h_{ij} - \cancel{\partial_i \partial_j h^k_k}) + R^{(global)}_{ij}$ indices: i, j, k = 1, 2, 3

$$- \partial_k\, \partial^k\, h_{ij} = \Box_{(4)}\, \bar{h}_{ij} = 2\, R^{(global)}_{ij} = - k_{Uni}^2 \cos(k_{Uni}\, t)\, \delta_{ij}$$

$\boldsymbol{R_{ij} \approx - k_{Uni}^2 \cos(k_{Uni}\, t)\, \delta_{ij}}$

$R_{pq} \approx \frac{1}{2} (\cancel{\partial_r \partial_p \bar{h}^r_q} + \cancel{\partial_r \partial_q \bar{h}^r_p} - \partial_r\, \partial^r\, \bar{h}_{pq} - \cancel{\partial_q \partial_q \bar{h}^r_r}) + R^{(global)}_{pq}$ indices: r, q, p = 4, 5, 6

$$- \partial_r\, \partial^r\, \bar{h}_{pq} = \Box_{(4)}\, \bar{h}_{pq} = 2\, R^{(global)}_{pq} = - k_{Uni}^2 \cos(k_{Uni}\, t)\, \delta_{pq}$$

$\boldsymbol{R_{pq} \approx - k_{Uni}^2 \cos(k_{Uni}\, t)\, \delta_{pq}}$

$\boldsymbol{R = g^{ij} R_{ij} + \gamma^{pq} R_{pq}}$

The field equations describe visible and hidden components of matter as complementary curvatures. Together, these two components generate the complete 7-dimensional curvature. It is shown that photons are locally null-geometric entities, as in general relativity, but globally follow a universal deviation.

15. Group Theory for FSM

The group theory of the FSM classifies the symmetries of rotations in the compact dimensions of the wave-field $F_{4\text{-}6}$ in order, among other things, to explain the strong force as an SU(3) symmetry arising from rotations in the dimensional planes $D_{45/46/56}$. Furthermore, the particle spin modes arising from the internal rotation of multiple fions in the bundle are to be classified as symmetry classes (e.g., SO(3)), and the modes are to be classified topologically. In doing so, both trivial and complex topologies with invariants for stability are addressed. From the FSM arise the groups derived from the geometry of cavity oscillations with their rotational representation

<u>The strong interaction (strong force) resulting from the rotations parallel to $D_{45/46/56}$:</u>

The 3-dimensional space in the wave-field with D_a has the indices: $a = 4, 5, 6$. The strong force is generated in this space. The strong force arises from the binding of quarks as fion bundles with other quarks. Exchange fions are the carriers and initiators of field exchange between quarks. The field exchange via exchange fions and the oscillation of quarks are explained by rotations between them. The metric component γ_{ab} in D_a has an SO(3) symmetry with a rotation group in $D_{45/46/56}$, but due to the fion rotations of multiple fions in the bundle, it becomes SU(3), which forms a special unitary group.

The rotation tensor ω_{ab} describes the symmetry of the internal rotations of the fibers in the bundle. For $N = 3$, it describes the strong force as an SU(3) symmetry.

Metrics in the compact wave-field:

$\gamma_{ab} = \delta_{ab}$

An infinitesimal rotation in the compact dimensions transforms the coordinates as follows:

$y^a + \delta y^a = y^a + \omega^a_b\, y^b$

The rotation tensor is antisymmetric:

$\omega_{ab} = - \omega_{ba}$

The compact metric γ_{ab} transforms as a (0, 2) tensor:

$\delta\gamma_{ab} = \partial_a(\delta y^c)\, \gamma_{cb} + \partial_b(\delta y^c)\, \gamma_{ac}$ with: $\partial_a(\delta y^c) = \omega_a^c$ results:

$\delta\gamma_{ab} = \omega_a^c\, \gamma_{cb} + \omega_b^c\, \gamma_{ac}$

- ω_b^a – Rotation tensor, antisymmetric; generator of the rotation; generates the invariance; dimensionless, because: $\partial_a(\delta y^c)$
- $y_a = R \cos(kt + \beta)$

An infinitesimal rotation in the compact dimensions causes a change in the metric. Using the subscripts and the relation $\gamma_{ab} = \delta_{ab}$, this leads to:

$\delta\gamma_{ab} = \omega_{ac}\, \gamma_b^c + \omega_{bc}\, \gamma_c^a$ → **general transformation formula** (2.141)

Applying the flat metric $\gamma_{ab} = \delta_{ab}$:

$\delta\gamma_{ab} = \omega_{ab} + \omega_{ba}$, because $\delta\gamma_{ab}$ is **antisymmetric**:

$\delta\gamma_{ab} = \omega_{ab} + (- \omega_{ba}) = 0$ → the **metric** γ_{ab} remains **invariant** under rotation.

General form of the rotation tensor depending on the symmetry class N:

$\omega_{ab}^{(N)} = \sum_{m=1}^{N} \Phi_m\, (G_m)_{ab} \cos(kt + \beta)$ (2.142)

- $\omega_{ab}^{(N)}$ – rotation tensor; describes the symmetry of the internal rotations of the fibers in the bundle
- G_m – generators of the Lie algebra of the group SO(N) or SU(N) with compact indices: a, b, c = 4, 5, 6; dimensionless
- Φ_m – infinitesimal angle of rotation; forms the general SU(3) transformation; dimensionless
- $\cos(kt + \beta)$ – disturbance component derived from the wave-field dimensions
- (kt) – phase angle, finite, for real oscillations of compact dimensions

- Field exchange between the wave field and the particle-field at $kt = 0$ under:
  Strong interaction: $kt = 0$; $\beta = 0$
  Weak interaction: $kt = 0$; $90° > \beta > 0$
  No interaction: $kt = 0$; $\beta = 90°$

The group SU(N) is the group of all unitary $N \times N$-matrices with determinant 1. The corresponding Lie algebra su(N) has dimensions.

a) Generators - classic:

$\text{Dim}(su(N)) = N^2 - 1 = 3^2 - 1 = 8$ → 8 generators

The rotation tensor is defined by the sum over the eight independent generators λ_m.

$$\omega^{(3)}_{ab} = \sum_{m=1}^{8} \Phi_m (\lambda_m)_{ab} \cos(kt + \beta) \quad (2.143)$$

- $\omega^{(3)}_{ab}$ – explained for $N = 3$ the SU(3)-symmetry with the strong force
- λ_m – Gell-Mann matrices that are exactly hermitian and trace-free; 3×3-matrices; spans the Lie algebra su(3)

b) Generators – FSM:

The geometry of the FSM provides a 6-dimensional field-space. According to the photon subspace theory (point 14), this space contains several 4-dimensional rotational orbits. Between particle-field $F_{1\text{-}3}$ and wave-field $F_{4\text{-}6}$, there is a total of 15 such rotational paths. **Table 3.1** in **Chapter 3.2** shows the possible dimensions spanned.

This results in su(4) with 15 generators:

$4^2 - 1 = 15$

The rotation tensor is calculated by summing over the 15 independent generators G_m.

$$\omega^{(4)}_{ab} = \sum_{m=1}^{15} \Phi_m (G_m)_{ab} \cos(kt + \beta) \quad (2.144)$$

With: $G_m = (J_{pq})_{ij} = \delta_{ip}\,\delta_{jp} - \delta_{iq}\,\delta_{jp}$

- +1 in position (p, q)
- -1 in position (q, p)
- all other entries are zero
- G_m – generators

The generators are 4 × 4 Hermitian, trace-free matrices. This form represents the complex Lie algebra (su(4)). Each matrix is antisymmetric and has only two nonzero entries. G_1 through G_8 (pairs within $D_{1\text{-}3}$ and $D_{4\text{-}6}$, respectively) correspond to the embedded SU(3) generators. G_9 through G_{15} are mixed generators and contain the connections between the visible $D_{1\text{-}3}$ and compact $D_{4\text{-}6}$, as well as exclusively the compact dimensions $D_{4\text{-}6}$.

Strong force:

$$G_m = \begin{pmatrix} \lambda_m & 0_{3x1} \\ 0_{1x3} & 0 \end{pmatrix}, \; m = 1,\dots, 8 \quad (2.145)$$

$G_1 = J_{1,2}$ (Rotation in a plane 1-2)

$$G_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$G_2 = J_{1,3}$ (Rotation in a plane 1-3)

$$G_2 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$G_3 = J_{1,4}$ (Rotation in a plane 1-4)

$$G_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$G_4 = J_{1,5}$ (Rotation in a plane 1-5)

$$G_4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$G_5 = J_{1,6}$ (Rotation in a plane 1-6)

$$G_5 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$G_6 = J_{2,3}$ (Rotation in a plane 2-3)

$$G_6 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$G_7 = J_{2,4}$ (Rotation in a plane 2-4)

$$G_7 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$G_8 = J_{2,5}$ (Rotation in a plane 2-5)

$$G_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Weak force and invisible (dark) matter:

$$T^k_{real} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0_{3x3} & e_k \\ e_k^T & 0 \end{pmatrix} \qquad k = 1, 2, 3 \qquad T^k_{imag} = \frac{i}{\sqrt{2}} \begin{pmatrix} 0_{3x3} & -e_k \\ e_k^T & 0 \end{pmatrix} \tag{2.146}$$

$G_9 = J_{2,6}$ (Rotation in a plane 2-6)

$$G_9 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$G_{10} = J_{3,4}$ (Rotation in a plane 3-4)

$$G_{10} = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$G_{11} = J_{3,5}$ (Rotation in a plane 3-5)

$$G_{11} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$G_{12} = J_{3,6}$ (Rotation in a plane 3-6)

$$G_{12} = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$G_{13} = J_{4,5}$ (Rotation in a plane 4-5)

$$G_{13} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$G_{14} = J_{4,6}$ (Rotation in a plane 4-6)

$$G_{14} = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Electromagnetic force:

$G_{15} = J_{5,6}$ (Rotation in a plane 5-6)

$$G_{15} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

→ 15. The generator is the remaining U(1) freedom. It explains U(1) as an emergent Abelian symmetry arising from 7-dimensional geometry and the gauge potential. In the FSM, it plays the role of the electromagnetic force.

SU(4) describes the full symmetry of all rotational orbits and contains SU(3) as a natural subgroup. The breaking of

SU(4) → SU(3) × U(1)

contains exactly 9 unbroken generators (8 from SU(3) + 1 from U(1)). The remaining 6 mixed generators, G_9 through G_{14}, are either completely or significantly weakened by the breaking mechanism. These generators mediate the coupling between the visible SU(3) block and the hidden part of the block. The breaking mechanism is geometric-dynamic and is modeled by the oscillating metric factor $\cos(kt + \beta)$.

This yields the phase angle β and naturally explains the separation of strong (visible) and weak/invisible (invisible, dark matter) forces, as well as the electric force.

- $\beta = 0$ and $\cos(kt + \beta) = 1$: perfect representation of the field exchange on the D_{56} dimensional level; results in full SU(4) symmetry → maximum strong force
- $\beta \neq 0$ or $\cos(kt + \beta) < 1$: mixed generators G_9 through G_{14} are modulated; this results in a violation of symmetry → weak force and transition to dark matter
- The refraction is natural and avoids the need for an additional Higgs mechanism.

Maximum number of N_{aF} for the 6-dimensional field space and projection:

The field-space is 6-dimensional and allows for 5 × 4-dimensional subspaces, which may have an overlap zone with their rotation paths at a single point. Accordingly, a bundle can contain only 5 active fions.

All five active fions generate a potential in the wave-field, although only three of them convey a partial charge with respect to the dimensional planes $D_{14/24/34}$. In the visible particle-field $F_{1\text{-}3}$, only the projection of the rotations onto the dimensional planes $D_{14/24/34}$ is registered. The rotations in the wave-field $F_{4\text{-}6}$ on the planes $D_{45/46/56}$ are invisible.

Resulting angular momentum L and projected spin:

The resulting **total angular momentum** is the sum of all fions involved in a bundle. This includes all active N_{aF} fions (up to 5 due to the 6-dimensional spatial geometry) and any temporary external N_{eF} that may be received.

$$L_{N\,fions}{}^2 = \sum_0^N \left(\frac{h}{4\pi}\right)^2_N \qquad \text{with: } N = N_{aF} + N_{eF} \qquad (2.147)$$

- L – angular momentum; unit: $\frac{\text{kg m}^2}{\text{s}}$
- N_{aF} – number of active fions in the bundle
- N_{eF} – number of external fions received temporarily

Spin is not an intrinsic property of the “particle” itself, but rather the geometric result of the rotations of the constituent fions in 6-dimensional field-space. The observed spin-½ fermions and spin-1 bosons arise inevitably from the number and binding configuration of the fions in the bundle.

In FSM, the quantization of spin is explained purely geometrically and follows directly from the 15 rotation generators of the compact dimensions. The total spin arises from the rotation of multiple fions in the bundle. These can consist of the number of internal bundle fions N_{aF} and additional (temporarily limited) connections of external fions N_{eF}, whereby only the SO(N) symmetry is registered, which reduces to SO(3) for the spin in the wave field $F_{4\text{-}6}$. The formula for the spin from rotations is:

$$S = \frac{1}{2}\,\frac{h}{2\pi}\,(N_{aF} + N_{eF})$$

$$S = \frac{h}{4\pi}\,(N_{aF} + N_{eF}) \qquad (2.148)$$

- S – spin; normalized to $\frac{h}{2\pi}$; dimensionless; determines intrinsic angular momentum; half-integer with $S = \frac{1}{2}$ for fermions, integer with S = 1 for bosons
- $\frac{1}{2}$ – Factor; accounts for the half-integer nature of the rotational speed $V_{Rot} = c$ by setting the maximum speed to $\frac{c}{2}$ for the single active fion as an internal fion within the bundle; prevents a variance > c in case of deviation; the fastest repetition of the period generates the next multiple with $\frac{c}{2}$
- $\frac{h}{2\pi}$ – reduced Planck constant; unit: Js; determines the quantum scale

- N_{aF} – number of fions in the bundle; N_{aF} ε N; dimensionless; determines the symmetry class; N_{aF} = 1 leads to spin $\frac{1}{2}$, N_{aF} = 2 leads to spin 1, N_{aF} = 3 leads to spin $\frac{1}{2}$, etc.

Examples of the electromagnetic and strong interactions:

The electron is the basic unit of **electrical interaction**. Electrical interaction occurs when an electron receives an external fion $f > f_{min}$ and uses it to exchange energy with another electron that also possesses an external fion. The total angular momentum results from the four individual angular momenta, each of which has a rotational velocity of $\frac{c}{2}$. In this case, the electron's spin is briefly bosonic (S = 1) for the duration of the interaction. The external fion is registered in the particle-field solely as an electric force. After the external fion is released, only the projection of the three active fions with effective spin S = $\frac{1}{2}$ remains in the particle-field.

In the case of the **strong interaction**, the electron also briefly possesses an externally received fion. The electron once again acquires four angular momentum components from N_{aF} and N_{eF}. As a result, the total spin temporarily becomes an integer again. This state triggers an interaction. In this case, the electron with 3 $\frac{1}{3}$ e^- reduces to 2 $\frac{1}{3}$ e^- to a 2/3 quark by splitting an active fion from the internal bundle into an exchange fion and a passive fion. The mechanics are explained in **Chapter 3.2**. The quark exists only because it forms a boson together with an exchange fion. The quark within it is, in this case, a 2 $\frac{1}{3}$ e^- electron in the wave-field. Exchange fions mediate the strong interaction. The total spin in the particle-field becomes a spin $S = \frac{1}{2}$ because, in the projection of the bundle, only 2 active fions and the external fion, each with spin $\frac{1}{2}$, remain.

In the case of a 1/3 quark, it interacts with another particle that may belong to the category of invisible (dark) matter.

The projection of a fermion onto the $D_{14/24/34}$ dimensions is always associated with spin $S = \pm\frac{1}{2}$. In the case of mesons, which consist of two fermions, the spin is $S = \pm\frac{1}{2} \pm \frac{1}{2}$. And in the case of baryons, which consist of three fermions, the spin is e.g. $3\,S = \pm\frac{1}{2} \pm \frac{1}{2} \pm \frac{1}{2}$.

<u>Charge and potential from recombination in a bundle:</u>

Using the potential from a bundle of bound fions

$$A^a_{eff,\,0} = \frac{Q}{4\pi\,\epsilon_0\,R}\,\cos(kt + \beta)\,\delta^a_4\,\delta^0_\mu\,\frac{c}{V_5} \qquad \text{with: } D_{56} \text{ – plane, } a = 6 \qquad (2.69)$$

the charge Q results from the position relative to the D_{56} (positive above, negative below). Although the invisible compact rotational orbits rotate asymmetrically parallel to the fourth dimension against the electrical potential of the universe, they possess only a periodically increasing and decreasing potential, which goes unnoticed by the particle-field. This applies to the rotational orbits along the dimensional planes $D_{45/46/56}$. The visible component acts differently. Only the visible components can impart a charge in the particle field $F_{1\text{-}3}$. This is because they immediately exchange their field within the dimensional planes $D_{14/24/34}$ as soon as the point of contact in the dimensional plane D_{56} is reached. Consequently, there can only be three partial charges, independent of particle complexity, which exists within the dimensional planes $D_{14/24/34}$. Other active fions that cross the dimensional plane D_{56} do not convey a partial charge due to their geometric position. However, they contribute to the total mass by having their share not only divided into thirds, but also into quarters or fifths.

Fundamental potential in the wave-field from a 4-dimensional perspective:

$$\phi = \frac{\pm Q}{4\pi\, \epsilon_0\, R} \cos(kt + \beta)$$

- ϕ – potential; unit: V; results from the reduction gradient
- Q – charge; unit: C; causes the strength; $\pm e$ from a position above or below the dimension plane D_{56}
- $4\pi\, \epsilon_0$ – constant; unit: $\frac{\mathrm{F}}{\mathrm{m}}$; results in Coulomb scaling
- R – radius; unit: m; creates distance
- $\cos(kt + \beta)$ – oscillation; creates the dynamic

The oscillating potential thus averages out to zero. However, the amplitude (maximum value) remains unchanged, which imparts an effective charge to the particle-field $F_{1\text{-}3}$ for active fions with the rotational orbits $D_{14/24/34}$.

$\cos(kt)_{amplitude} \rightarrow 1$ (acts effectively as a Q charge in 4D)

Static result:

$$\phi = \frac{\pm Q}{4\pi\, \epsilon_0\, R} \tag{2.149}$$

Total load divided into partial charges according to the active fions:

$$Q = \frac{e}{3} \sum_{i=1}^{3} \delta_{i,\text{coupling}} = N_{aF}\, \frac{e}{3} \qquad N_{aF}\ \varepsilon\ \mathrm{N} \tag{2.150}$$

- Q – electric charge with $[Q] = \mathrm{As} = \mathrm{C}$
- e – electric charge of the elementary particle electron
- $e = 1{,}6022\ 10^{-19}$ C
- N_{aF} – number of active fions in the dimensional planes $D_{14/24/34}$, at most 3

Note: When incorporating the compact dimensional planes $D_{45/46/56}$ into the shaft field, there is a limit of 5 possible rotational paths.

Quark charge:

Quark types arise from an original electron structure with varying numbers of active fions. Quarks never exist as individual particles on their own. In the FSM, they are part of a boson, which consists of a quark and its exchange particle. The exchange fion is created by the reduction of an active fion in the electron, which carries a partial charge in the particle-field. The sign is determined by the configuration in the D_{56} dimensional plane, indicating whether the rotation occurs above or below.

$$Q = \pm(3 - N_{F/A})\,\frac{e}{3} \tag{2.151}$$

- $\frac{e}{3}$ – one-third of the base charge; unit: C; one-third of the 3D wave-field (SU(3)) symmetry, implicitly
- $N_{F/A}$ – Number of active fions resulting from conversion to exchange fions/passive fions, or those that do not generate a potential in the wave-field relative to the dimensional plane D_{56}

Equation (2.151) explains fractional charge geometrically in relation to D_{56}.

Possible elementary charges:

Electron: $Q = -\,(3 - 0)\,\frac{e}{3} = -\,e$

Positron: $Q = +\,(3 - 0)\,\frac{e}{3} = +\,e$

Neutrino: $Q = \pm\,(3 - 3)\,\frac{e}{3} = 0$

u/d-Quarks: $Q = \pm\,(3 - 1)\,\frac{e}{3} = \pm\,\frac{2}{3}e$

u/d-Quarks: $Q = \pm\,(3 - 2)\,\frac{e}{3} = \pm\,\frac{1}{3}e$

C-Quarks: $Q = \pm\,(3 - 1)\,\frac{e}{3} = \pm\,\frac{2}{3}e$

C-Quarks: $Q = \pm\,(3 - 2)\,\frac{e}{3} = \pm\,\frac{1}{3}e$ (hypothetically possible)

S-Quark: $Q = \pm\,(3 - 2)\,\frac{e}{3} = \pm\,\frac{1}{3}e$

S-Quark: $Q = \pm (3 - 1)\,\frac{e}{3} = \pm\frac{2}{3}$e (hypothetically possible)

B-Quark: $Q = \pm (3 - 2)\,\frac{e}{3} = \pm\frac{1}{3}$e

B-Quark: $Q = \pm (3 - 1)\,\frac{e}{3} = \pm\frac{2}{3}$e (hypothetically possible)

T-Quark: $Q = \pm (3 - 2)\,\frac{e}{3} = \pm\frac{1}{3}$e (hypothetically possible)

T-Quark: $Q = \pm (3 - 1)\,\frac{e}{3} = \pm\frac{2}{3}$e (hypothetically possible)

Trivial and complex topology for mode classification:

Topology determines whether modes (such as particle states) remain stable or decay. In the FSM, the compact wave-field dimensions D_4, D_5, D_6 are topologically structured. Modes (Fourier expansion $e^{\frac{iny^a}{R}}$ are stable oscillations whose stability depends on the topology of the compact manifold:

- Trivial topology: simple, stable modes (n is an integer, no holes); z.B. S^1 = circle
- Complex topology: greater stability due to holes/genus, but potential instability at high complexity; e.g. T^3 = 3-torus (Calabi-Yau)

First invariant n - number of turns:

The simplest topological classification is the winding number n derived from the Fourier expansion of the modes.

$\phi(x, y) = \sum_n \phi_n(x) e^{\frac{iny^a}{R}}$ with: $-\infty < n < \infty$

Real equivalent:

$\phi(x, y) = \sum_{n=1}^{3} \phi_n(x) \cos(\frac{ny^a}{R})$ with: n = 1, 2, 3

n is a topological invariant for the number of windings around the compact dimensions. $n = 0$ for the constant mode (hypothetically massless). $n \neq 0$ for massive modes ($m \sim \frac{n}{R}$). This is expressed in **Chapter 3.5** as a “family of dimensions”.

Second invariant χ – Euler characteristic:

χ classifies stability. The general topological classification uses the Euler characteristic χ.

$X = 2 - 2g$

- X – Euler characteristic; dimensionless; yields topological invariants
  $X = V - E + F$
  V – vertices
  E – edges
  F – areas in triangulation
  - $X = 2$ for ball S^2 (stable but not compact in FSM)
  - $X = 0$ for torus T^2 (trivial, stable for photons)
  - $X < 0$ for higher-order functions (complex, instability possible)
- g – genus; g ε N; dimensionless; determines the number of holes in the compact manifold
  - g = 0 for S^2
  - g = 1 for T^2
- General:
  $X = 0$; trivial, for stable modes (photons, stable visible particles)
  $X \leq 0$, $g \geq 1$; complex, for unstable or hidden (dark) modes, or greater stability through holes/genus

Classification:

- The maximum number of active fions in a bundle is 5, due to the 6-dimensional nature of the field-space.
- Trivial topology:
  - S^1 (Circle); g = 0; $X = 2 - 2 \cdot 0$; but often in FSM $X = 0$ for an effective torus
  - n – an integer
  - Stable visible modes: photons, light fermions with $n = 0$, stable particles
- Complex topology:
  - T^3 (3-torus); g = 1 per dimension; $X = 0$; invariants such as the Chern class $c_1 = 0$
  - Stable modes: invisible massive modes (dark matter)
  - Calabi-Yau (complex 3-manifold); $X = -200$ for typical sextics; invariants c_2, c_3
    Stable modes: higher generations (C-, B-, and T-quarks); stability via $V(\phi_n)$ and metric oscillation $\cos(kt + \beta)$
  - The dimension reduction factor (DimFactor) – **Chapter 3.5** – applies to complex bundles with five fions, e.g., B-quarks, in order to keep their resulting rotational velocity at $V_{Rot} \leq c$.
- Additional classifications:
  - Homotopy groups for T3 (coefficient of winding)
  - Chern classes c_1, c_2, c_3 for complex manifolds
  - Stability through $V(\phi)$; stability through R

Trivial stable modes correspond to visible particles, while complex modes correspond to hidden (dark) matter and higher generations.

16. Chern classification in FSM

The Chern classification is a topological invariant for complex manifolds in the compact wave-field dimensions $D_{4\text{-}6}$. It serves as a mathematical "proof" of correctness, as it provides a geometric justification for anomaly-free properties, the number of generations, and interactions. It shows that 3-dimensional wave-field geometry enforces the observed symmetries (e.g., three generators, anomaly-free) without resorting to supersymmetry. The derivation follows from compactification and the metric, with the Chern classes serving as invariants for stability and anomalies. More precisely, the Chern classes c_k (k = 1, 2, 3) are cohomological invariants for vector bundles over complex manifolds, which in the FSM prove the stability of the modes (n = 1, 2, 3) and anomaly-free nature.

General formula for Chern classes:

$$c_k(E) = \det(1 + \frac{\mathrm{i}}{2\pi} F) = \frac{1}{k!} (\frac{\mathrm{i}}{2\pi})^k \,\mathrm{Tr}\,(F)^k \tag{2.152}$$

- $c_k(E)$ – k-th Chern class of the vector bundle E; dimensionless; defines invariants for the bundle topology; k corresponds to the rank of the wave-field, where k = 1, 2, 3; E – tangent bundle of the manifold
- $\frac{1}{k!}$ – factor from the determinant expansion
- $(2\pi)^k$ – normalization; dimensionless; induces a cohomology class in $H^{2k}(M, \mathrm{Z})$; Z – the set of integers
- i – symbol for an imaginary form; provides a hermetic and real trace, which is crucial for the physical interpretation
- Tr – trace; dimensionless; causes contraction via indices
- F – curvature form (2-form); unit: $\frac{1}{\mathrm{m}^2}$; results in a field strength of A_μ^a
- $(\frac{F}{2\pi})^k$ – power; unit: $(\frac{1}{\mathrm{m}^2})^k$; results in higher moments of curvature

<u>Chern class c_1 :</u>

The first Chern class is defined as:

$$c_1(E) = \frac{\mathrm{i}}{2\pi}\,\mathrm{Tr}\,F$$

where the Chern classes arise from the curvature:

$$F = \mathrm{d}A + A \wedge A \text{ (aus } A_\mu^a)$$

The field strength F corresponds exactly to the Chern curvature of the gauge bundle:

$F^a = \mathrm{d}A^a + f^{abc} A^b \wedge A^c$

- f^{abc} – totally antisymmetric structure constants of the Lie algebra; defines the gauge groups

or in component form:

$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu$

with F: F – is the 2-form

$F = \frac{1}{2} F^a_{\mu\nu}\, \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu\, T^a$

- c_1 – first Chern class, dimensionless; determines charge topology; 0 denotes anomaly-free models
- $\frac{\mathrm{i}}{2\pi}$ – normalization; dimensionless; returns integer values
- Tr F – curvature radius; unit: $\frac{1}{\mathrm{m}^2}$; the integral over manifolds
- Tr – curvature trace via the group index a; for SU(N) or U(1)
- F – is the 2-form
- i – imaginary form; ensures real and integer values for c_1

In FSM, the following applies:

Tr $F = 0$

because:

- the T^a generations of SU(4) trace-less → Tr $T^a = 0$.
- the Abelian trace (U(1) part) is absent in the non-Abelian group or is suppressed by the geometry.
- the charge does not arise globally (no monopoles), but solely from the wave-field dynamics and the geometry of the dimensional plane D_{56}.

Therefore:

$$c_1(E) = \frac{i}{2\pi} \,\mathrm{Tr}\, F = 0 \qquad (2.153)$$

<u>Chern class c_2:</u>

General:

$$c_2(E) = \frac{1}{2!} \left(\frac{\mathrm{i}}{2\pi}\right)^2 \mathrm{Tr}\,(F)^2 = \frac{1}{4\pi^2}\frac{1}{2}\,\mathrm{Tr}\,(T^a\, T^b) = \frac{1}{8\pi^2}\,\mathrm{Tr}\,(F \wedge F)$$

In FSM, a complex 3D manifold applies to rotation in the dimensional planes $D_{45/46}$:

$$F = \mathrm{d}A + A \wedge A = \frac{1}{2} F^{a}_{\mu\nu}\, \mathrm{d}x^{\mu} \wedge \mathrm{d}x^{\nu}\, T^{a}$$

$$F^{a} = \mathrm{d}A^{a} + f^{abc} A^{b} \wedge A^{c}$$

For non-Abelian groups, the following important property holds:

$\mathrm{Tr}(F \wedge F) = \mathrm{Tr}((\mathrm{d}A + A \wedge A)^2)$

Squaring of F :

$F \wedge F = (\mathrm{d}A + A \wedge A) \wedge (\mathrm{d}A + A \wedge A)$

Distribution of the wedge product (∧ is associative and antisymmetric):

$F \wedge F = \mathrm{d}A \wedge \mathrm{d}A + \mathrm{d}A \wedge (A \wedge A) + (A \wedge A) \wedge \mathrm{d}A + (A \wedge A) \wedge (A \wedge A)$

Summary:

$\mathrm{Tr}(\mathrm{d}A \wedge (A \wedge A)) = \mathrm{Tr}((A \wedge A) \wedge \mathrm{d}A)$ both terms of equal size

$2\mathrm{Tr}(\mathrm{d}A \wedge (A \wedge A))$

Results in:

$\mathrm{Tr}(F \wedge F) = \mathrm{Tr}(\mathrm{d}A \wedge \mathrm{d}A) + 2\mathrm{Tr}(\mathrm{d}A + A \wedge A) + \mathrm{Tr}(A \wedge A \wedge A \wedge A)$

The running integral $\int_M$ over $\mathrm{Tr}(F \wedge F)$ results in:

$\int_M \mathrm{Tr}(\mathrm{d}A \wedge \mathrm{d}A) = 0$ (exactly)

$\int_M 2\mathrm{Tr}(\mathrm{d}A + A \wedge A) = 0$ (exactly)

$\int_M \mathrm{Tr}(A \wedge A \wedge A \wedge A) = \mathrm{Tr}(F \wedge F)$ (not exact and not closed in general; this one precisely measures the topological twist of the bundle)

- $\mathrm{Tr}(A \wedge A \wedge A \wedge A)$ arises from the non-Abelian self-coupling of the gauge fields A^{a}_{μ}
- In FSM, self-coupling arises geometrically from the curvature of the wave-field dimensions, particularly the deviation from the dimensional plane D_{56}

Result:

$$c_2(E) = \frac{1}{8\pi^2} \int_M \mathrm{Tr}\,(F \wedge F) \tag{2.154}$$

- $\int_M$ – integral; represents the underlying 4-dimensional manifold for the compact space in the wave-field; measures the topological twist of the gauge bundle across the wave field dimensions
- $F \wedge F$ – the wedge product of the field strength in 4-form; the trace measures the topological twist of the calibration bundle, resulting in an integer factor; domain $D_{45/46}$
- $c_2(E) \neq 0$ creates local fluctuations that stabilize the binding of quarks (as 3-fion bundles).
- In FSM, k is chosen such that the index = 3 for the chiral generations.

Chern class c_3:

General:

$$c_3(E) = \frac{1}{3!} \left(\frac{i}{2\pi}\right)^3 \text{Tr } (F)^3 = \frac{1}{6} \frac{1}{8\pi^3} \text{Tr } (F \wedge F \wedge F)$$

In FSM, a complex 3D manifold applies to rotation in the dimensional planes $D_{45/46/56}$:

$$c_3(E) = \frac{1}{24\pi^3} \int_M \text{Tr } (F \wedge F \wedge F) = 0 \quad (2.155)$$

- c_3 – Determinant component for the third class; measures curvature; in FSM, $F \sim k \cos(kt)$ is diagonal, so $\det(F) = 0$, since rotations in a 3D wave-field are completely antisymmetric ($D_{45/46/56}$); antisymmetric matrix consisting of: $\omega_{ab} = \omega_{ba}$
- $(F \wedge F \wedge F)$ – The wedge product of the field strength in 6-form; the trace measures the topological twist of the calibration bundle, which generally results in an integer factor; domain $D_{45/46/56}$
- $\text{Tr } (F \wedge F \wedge F) = 0$, because the product of three trace-zero matrices has trace zero; (in SU(N), $N \geq 2$); $f^{abc} \neq 0$ causes the $\text{Tr}(F^3)$ signal to disappear; for SU(N) with $N \geq 2$ is the trace of three generators zero
- The compact manifold has no non-trivial 6-cohomology in the relevant dimension; for 3D compact manifolds → no 6-form integrals exist
- Although the D_{56} deviation generates effective $SU(2)_L$ chirality, it does not generate a non-trivial c_3 factor
- Chirality arises solely from dynamic geometry (D_{56}- angle deviation, frequency shift, $\cos(kt)$), but not from a topological c_3 factor
- $c_3 = 0$ follows directly from the fact that the calibration group is non-Abelian ($\text{Tr } (F^3) = 0$) → The integral vanishes trivially.

Insights from the results of the Chern classes:

1. **Charge:**
   - $c_1 = 0$ means that the FSM does not require classical monopoles; the charge is not global, but arises purely from wave-field dynamics
   - Each generation n has an effective charge Q_n that arises from the geometry of the dimensional plane D_{56}; charge topology:
     $\sum_{gen} \text{Tr}\, Q^3 = 0 \rightarrow Q_1^3 + Q_2^3 + Q_3^3 = 0$
   - Force load structure:
     $n = 1$ (1. Generation): $Q = \pm 2/3e$; $Q = \pm 1/3e$; $Q = \pm e$;
     $n = 2$ (2. Generation); $n = 3$ (3. Generation): analog structure
     Sums over all quarks + Leptons per generation: Tr $Q^3 = 0$
     Possible combination: $\frac{2}{3} \cdot 3$ „color" + $(-\frac{1}{3} \cdot 3) + (-1) + 0 = 0$
     The charge combination can be reflected accordingly
2. **The Three Generations:**
   - ($n = 1, 2, 3$) must be free of anomalies when taken together
   - No gauge anomalies in SU(2) × U(1), as three generations balance the loads.
   - There are exactly three generations in the 7-dimensional FSM model.
   - Anomaly cancellation, since c_1 is an integral over the anomaly polynomial.
3. **Weak interaction:**
   - $c_1 = 0$ for global freedom, but locally $F \neq 0$ allows deviations for the F curvature in $D_{45/46}$
   - Weak strength SU(2) from rotation to $D_{45/46}$ with 2D, deviation from SO(3) due to its complex structure.
   - $c_2 = \frac{1}{4\pi^2} \text{Tr}\, F^2 \neq 0$ measures potential local "distortion," creating local deviations that stabilize the bond
   - Weak interactions can now arise only through geometric deviations of the metric γ_{ab} in 2D subspaces (relative to the dimensional plane D_{56}); F as a measure
   - The weak interaction is depicted as a "shifted" rotation (parity violation due to the deviation), since $F \wedge F$ is antisymmetric (chiral); geometrically demonstrated by $c_1 = 0$
4. **Strong interaction (strong force):**
   - $c_1 = 0$ ensures the absence of anomalies and implies that the SU(3) symmetry (for the strong interaction between quarks) arises from the trivial topology of $X = 0$
   - $c_2 \neq 0$ creates local fluctuations that stabilize the binding of quarks (as 3-fion bundles)
   - $c_3 = 0$ limits the number of generations to 3 and excludes higher modes within the 7D FSM framework

- Chern's results show that the strong force is not a separate force, but rather a geometric invariant of compact curvature. FSM offers added value because it explains QCD structures without quantization or supersymmetry, but with consequences such as anomaly-free baryon formation and "dark matter" as unstable modes.

5. **Electromagnetic interaction:**
   - $c_1 = 0$ ensures the absence of anomalies and implies that the U(1) charge (electric charge) arises as a trivial topology, $X = 0$
   - $c_2 \neq 0$ creates local variations that stabilize the photon polarization
   - $c_3 = 0$ limits electromagnetic interaction to a single generation and excludes higher modes within the 7D FSM framework
   - Chern's results show that the electromagnetic interaction is not a separate force, but rather a geometric invariant of compact curvature. FSM offers added value because it explains QED structures without quantization or supersymmetry, but with consequences such as anomaly-free photon propagation and dark energy arising from electromagnetic-like vacuum terms.
6. **No additional generations in 7D:**
   - $c_1 = 0$ for the tangent bundle, this implies a trivial bundle with no higher "windings"
   - In the 7D version of the FSM, the generation modes $n = 1, 2, 3$ arise from 3D compact wave-field dimensions; higher values of n require $c_1 \neq 0$ as an additional charge topology.
   - From 7D (4D visible + 3D invisible compact), the dimensionality of D = 3 imposes exactly three modes; the maximum is determined by the dimensions in the compact wave-field.
   - $c_3 = 0$ excludes higher generations, since $n = 4$ would require $c_3 \neq 0$
7. **Gravitational anomaly in 7D:**
   - $c_1 = 0$ does not imply a chiral anomaly, since anomalies ~ Tr F in odd dimensions such as 7D vanish → FSM is already consistent with both classical and quantum mechanics
   - Possible gravitational anomalies in D-dimensions arise from Tr $R^{D/2+1}$; for FSM D = 7 ~ Tr $R^{4,5}$; would only be relevant in the case of even D
   - $c_1 = 0$ for gauge bundles, this implies Tr $F = 0$; extended to Tr $R = 0$ for gravity
   - Consequently, FSM exhibits no gravitational anomalies in 7D; $c_1 = 0$ confirms freedom

Conclusion:

The Chern classification for the FSM model provides mathematical proof of its correctness based on its lack of anomalies and the presence of three generations. The FSM model is simple, does not require supersymmetry, and is predictable through the modes of topology. This creates a new framework for particle physics as an alternative to string theory, offering anomaly-free physics without extra assumptions and the potential to incorporate dark energy from topology.

17. Fine-structure constant in FSM

The fine-structure constant α is the dimensionless coupling constant of the electromagnetic interaction. It determines the threshold at which a photon can exchange its field with another particle. According to Sommerfeld, the relationship for the fine-structure constant is:

$$\alpha = \frac{e^2}{2\,\epsilon_0\,h\,c} \tag{2.156}$$

- α – Sommerfeld fine-structure constant; dimensionless; $\alpha = \frac{1}{137{,}065}$; scales the electromagnetic coupling
- e – elementary charge; unit: C; e = 1,602 10^{-19} C; causes the base charge
- ϵ_0 – electric field constant; unit: $\frac{As}{Vm}$; ϵ_0 = 8,8542 10^{-12} $\frac{As}{Vm}$; scales the Coulomb potential
- h – Planck's constant; unit: Js; h = 6,626 10^{-34} Js; for Quantum Scale
- c – maximum speed; unit: $\frac{m}{s}$; c = 299792458 $\frac{m}{s}$

The FSM will alternatively derive the reciprocal value of the fine-structure constant geometrically to enable verification. This is demonstrated using the general formula for coupling frequencies and object masses in **Chapter 3.7**, formula (3.34).

In group theory, it was explained that the electrical interaction occurs via an external fion, which must exceed the minimum coupling frequency. With the electron as the base particle with a factor of 1, the following ratio between the electron frequency and the minimum coupling frequency must hold in order to satisfy the condition for interaction with the lowest excitation:

$$\alpha = \frac{f_e}{f_{min}} \approx \frac{1}{137} \tag{2.157}$$

The FSM predicts that particle frequencies and masses can be modeled as multiples of the electron/positron, scaled by the fine-structure factor!

18. Spin-0-Pair Theory of FSM – Entanglement

Definition of Entanglement:

Quantum entanglement is a state in which two or more particles (or systems) are so interconnected that the quantum state of the entire system cannot be described as the product of the individual states, even if the particles are far apart.

- Measuring one particle immediately determines the state of the other, regardless of the distance between them.
- There is no standard explanation, such as for hidden variables or signals traveling faster than the speed of light.
- Violates Bell's inequalities, which have been experimentally confirmed.
- Observed in photon pairs, electron spins, fion traps, superconductors, diamond-NV-centers, and even in macroscopic systems containing over ~ 10^{12} atoms.

Assumptions used by the FSM model to explain entanglement:

1) Conservation of energy and particle-antiparticle symmetry

- The 7D action is invariant under charge conjugation (C) and parity (P) in the compact wave-field dimensions.
- The global component of the tensor $T_{MN}^{(\text{global})} = (1 + \cos(kt + \beta))\, \delta_{MN}$ is C-symmetric and always produces electrons and positrons in pairs.
- Number of electrons = number of positrons (exactly, except for the CP violation caused by an angular deviation from the dimensional plane D_{56} – Z-, W-, H- bosons; the underlying reason is illustrated in **Figures 3.24–3.26**)

2) The photon as a spin-1 field with two possible configurations of its rotational paths

- The direction of rotation for both photons is either clockwise or counterclockwise.
- Dimensional plane D_{56} separates the global potentials of the universe. It can be represented as the “equatorial plane” of rotation, which reflects the rotation in an antisymmetric manner relative to itself.
- Helicity +1: rotates above the dimensional plane D_{56}
- Helicity -1: rotates below the dimensional plane D_{56}

3) Dark energy as a spin-0-pair in the uncoupled state < f_{min}

- Before reaching the minimum coupling frequency fmin, matter exists as an unbound pair of two spin-1 photons originating from a common field body of a single oscillating wave packet relative to the dimensional plane D_{56}. **Figure 7.2** provides a possible illustration of the universe in its initial state.

- The two constituent photons have opposite helicities (+1 and -1) with a total spin of 0. Total angular momentum $S_{total} = 0 = S_1 + S_2$ (singlet)
- The shared field means that their wave functions are not separable. Even when assigned as positive or negative partial charges in the form of an active particle, they remain **entangled** with one another.
- A change in the helicity of one spin-1 particle (e.g., due to measurement or absorption) instantly alters the state of the other, leading to the classical observation of entanglement.
- Their electric potential is maximally effective internally but neutral externally. The two sub-photons are electrically attracted to one another. Only their globally determined gravitational potential forces them into a stable orthogonal configuration relative to the dimensional plane D_{56}. In other words: Dark energy is the carrier of the gravitational potential. With this configuration, the wave function prevents a classical approximation and, consequently, an annihilation reaction.

4) Transition at f_{min}

- As soon as the oscillation frequency reaches $f \geq f_{min}$, the common field body breaks apart
- The spin-0 state decays into two independent spin-1 photons, causing dark energy to become electrically coupled and thus “accessible”.

Consistency with the FSM model:

- $V(\phi_n)_{dark} = \frac{9}{4} k^4 (1 - \cos(kt + \beta))^2$ describes precisely the potential of the spin-0 pair, provided that $f < f_{min}$
- $T_{\mu\nu}^{(dark)} = -\frac{1}{2} g_{\mu\nu}^{(4)} (2V(\phi_n)_{dark})$ is the tensorially compactified 4D component of the momentum-energy tensor
- The longitudinal component: The longitudinal polarization of the massive spin-1 photon corresponds to the invisible, uncoupled component in the common field body.
- Conservation of energy: The spin-0-state has energy 2 E_{Pho}. After the decay, two photons, each with E_{Pho}, are conserved.
- D_{56}-deviation: The local rotation with the term $\cos(kt + \beta)$ relative to the dimensional plane D_{56} produces exactly the asymmetry between helicity +1 and -1.

Wave function for the spin-0-pair (before $f < f_{min}$):

$$|\Psi_{pair}\rangle = \frac{1}{\sqrt{2}} (|+1\rangle_1 \, |-1\rangle_2 - |-1\rangle_1 \, |+1\rangle_2) \otimes |\phi_{common}(x, y)\rangle \qquad (2.158)$$

- $|+1\rangle_1 \, |-1\rangle_2$ – asymmetric helicity states of the two photons
  +1 = projection of the rotation above the dimensionsal plane D_{56}
  -1 = projection of the rotation below the dimensional plane D_{56}
- $\frac{1}{\sqrt{2}}$ – The prefactor ensures antisymmetry under the exchange of the two photon components, which is necessary for bosons with a spin-0-total state.
- Subtraction ensures that the total spin in the helicity sum is 0. The total spin of 0 arises from the orbital angular momentum of the uncoupled field body.
- $|\phi_{common}(x, y)\rangle$ – common scalar field; described by:
  $V(\phi_n)_{dark} = \frac{9}{4} k^4 (1 - \cos(kt + \beta))^2$
  → The field body is …
  - a standing wave in the dimensional plane D_{56} that "carries" both photons.
  - **not separable before** $f < f_{min}$.
  - **unable to couple** before disintegration.
- **Entanglement:** The antisymmetric component of helicity ensures that measuring the helicity of the first photon instantly determines the helicity of the second, resulting in classical entanglement without signal transmission.

Modified Proca equation after the decay ($f \geq f_{min}$):

Once f_{min} is exceeded, the spin-0 field decays into two independent massive spin-1 photons. The two spin-1 photons become locally independent and electrically coupled. The effective vector field of photon A_μ then obeys the Proca equation for a massive vector field.

$$\square_{(4)} A^a_\mu - \partial_\mu (\partial^\nu A^a_\nu) + 4\pi^2 m_a^2 \frac{c^2}{h^2} A^a_\mu = 0 \quad \text{with: } m = \frac{n\,h}{2\pi\, c\, R} \quad \text{and: } n = 1, 2, 3 \quad (2.159)$$

- The term $m_a^2 A^a_\mu$ ...
  - is derived from the potential $V(\phi_n)$ after compactification and describes the effective mass of the longitudinal component.
  - breaks the gauge invariance and allows for a non-transverse (longitudinal) solution, which is physically real and carries mass.
- After the disintegration …
  - the scalar potential field changes from $V(\phi_n)_{dark}$ to $V(\phi_n)_{pot}$.
    $V(\phi_n)_{pot} = \frac{9}{4} k^4 \cos^2(kt + \beta)$
  - a longitudinal polarization is activated, which generates a mass with.

Physical consequence:

- Photon now carries a mass $m_{Pho} = \frac{n\,h}{2\pi\, c\, R}$

- Dispersion: $k^2 = w^2 c^2 + 4\pi^2 m_a^2 \frac{c^4}{h^2}$; (massive, $v < c$)
- Yukawa potential (effective interaction): $V(r) = \frac{e^{-mr}}{r}$ (2.160)
  - $V(r)$ – The Yukawa potential causes the interaction to decay exponentially over long distances.
  - r – distance between the source and the observation point
  - $r \geq 0$ – unit: m, fm, angstrom – depending on the context
  - $r = 0$ – refers to the location of the source itself (where the potential typically diverges)
  - r – In the context of nuclear and particle physics, this refers to the distance between objects, and is applied in **Chapter 3**, “Particle Model”.
  - r – In the context of cosmology, this refers to the field radius (event horizon) and is discussed in **Chapters 2.3** and **7**.
- Three polarizations for spin-0 field bodies: 2 transverse + 1 longitudinal
- After the disintegration:
  - **Transverse wave** remains with: $h_{\mu\nu} = 0$; mass = 0
  - **Longitudinal wave** is: $\square_{(4)} \phi_n = m_n^2 \ \phi_n$; mass dependent on mode n

Entanglement is described as a 7-dimensional geometric effect of the configuration relative to the dimensional plane D_{56}. This explains why electrons and positrons always arise in pairs, why dark energy is initially hidden, and why entanglement is non-local. The longitudinal component and the Yukawa attenuation are direct consequences of mass generation by the dark energy potential $V(\phi_n)_{dark}$. This property can be verified using the example of the universe (**Chapters 7.1–2**). Due to the fractional-free scaling, these properties are present at every quantization starting from the minimum coupling frequency f_{min}.

19. Scalability

Scalability is the ability to describe phenomena uniformly across all orders of magnitude (from the Planck-scale microcosm to the macrocosm of the universe) without any discontinuities. In the FSM, it arises from the geometric nature of matter as a relativistic oscillation, scalable by the field radius r and the angular frequency k. The FSM is supersymmetry-free, explicitly predicts masses, consists of a dynamic sine stabilization, and is testable.

<u>Scalability from the field equations:</u>

The field equations are scaled by the field radius r. The field radius is a characteristic quantity of matter that arises from its relativistic fields. The nominal value of r is specified at the origin of the inertial system. In relativistic terms, the nominal value r becomes $r(t)$. In the field equations, the field radius appears as follows:

$$\delta g \sim g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \sim \frac{GM}{c^2\, r} \quad \rightarrow \quad R_{MN} \sim \frac{1}{r^2} \quad \rightarrow \quad T_{MN} \sim \frac{c^4}{8\pi G r^2}$$

- $g_{\mu\nu}$ – metric perturbation; corresponds to the deviation of space-time from the flat Minkowski metric due to gravity; from point 1.
- $h_{\mu\nu}$ – disturbance term for gravitational waves
- $\frac{GM}{c^2\, r}$ – gravitational curvature strength; dimensionless
- R_{MN} – Ricci tensor; unit: $\frac{1}{\mathrm{m}^2}$
- r – field radius; unit: m; $r = \frac{GM}{c^2}$ (2.134)
- T_{MN} – Momentum-energy tensor; unit: $\frac{\mathrm{kg}}{\mathrm{m\ s}^2}$

Macroscopic gravity:

Metrical disturbance

$$h_{\mu\nu} = \frac{GM}{c^2\, r}\,[1 + \cos(kt + \beta)]$$

is applied directly to macroscopic masses M and radii r.

Global scale:

The global metric is:

$$\eta_{\mu\nu}\,[1 + \cos(k_{\mathrm{Uni}}\, t)]$$

The global oscillation $\cos(k_{\mathrm{Uni}}\, t)$ s still the same function as for elementary particles. As the field radius r, increases, the angular frequency k decreases.

The Chern classes $c_1 = 0$ and $c_3 = 0$ apply unchanged to the entire universe.

Scalability through sinusoidal rotation:

Rotation about $\cos(kt)$ with the angular frequency k

$$k = \sqrt{\frac{G\, M}{r^3}} \qquad (2.135)$$

shows the possible range of frequencies:

$$m_{obj} = \frac{h\, c^2}{G\, \{m_{obj}\, k_{obj}\}\, \lambda_{obj}} = \frac{h}{c\, \lambda_{obj}} = \frac{h\, f_{obj}}{c^2} \qquad (2.192)$$

Scalability through universal constants identified by FSM (**Chapter 2.3**):

Space-time constant: $c = k\,r \rightarrow k = \frac{c}{r}$ (2.136)/(2.174)

$$c = 299792458\,\frac{\text{m}}{\text{s}}$$

Mass time constant: $m\,k = 4{,}0396\;10^{35}\,\frac{\text{kg}}{\text{s}}$ (2.175)

Mass-space constant: $\frac{m}{r} = 1{,}34746\;10^{27}\,\frac{\text{kg}}{\text{m}}$ (2.176)

Scalability based on Planck's constant, according to the formula (2.185):

$h = \lambda_{obj}\,r_{obj}\,m_{obj}\,k_{obj} = \lambda_{obj}\,m_{obj}\,c$ $[h]$ = Js with: $c = k\,r$

Scaling of energy according to formulas (2.180) and (2.182):

$E = r_{obj}\,m_{obj}\,k_{obj}\,c$ with: $c = k\,r$

$E = m_{Obj}\,r_{obj}^2\,k_{obj}^2 = m_{obj}\,c^2$

$E = m_{Obj}\,G\,\{m_{obj}\,k_{obj}\}\,\frac{1}{c}$

Scalability through frequencies:

$$\alpha = \frac{f_e}{f_{min}} \approx \frac{1}{137} \qquad (2.157)$$

In the general formula for particles, the existing relativistic rotations are accounted for within the wave-field geometry.

$$f_{obj} = \frac{1}{2}\,(\text{BK}\,(\text{KK})^3)^n \cdot \text{TK} \cdot \text{DimFactor} \cdot f_e \qquad (3.27)$$

Why the FSM model scales so well:

Uniform geometry: All scales use the same 7D metric, the same rotation tensor $\omega_{ab}^{(4)}$ and the same oscillation function $\cos(kt + \beta)$.

No new parameters: Only the values of R, n r, k, β; the basic equations remain the same.

Projection principle: What appears in the wave-field as a spin-0 pair or complex topology always projects onto the observed spin - $\frac{1}{2}$ - fermions and spin-1 bosons in the particle-field.

Topological invariants: The generation number/winding number n, the Euler characteristic X, and the Chern classes c_k are scale-invariant and explain stability from the Planck scale to the universe.

20. Comparison of FSM with classical models and string theory

To conclude this **Chapter 2.2**, we will provide a brief overview of the key advantages of the FSM model compared to previous theories.

Unification of all forces into a single model:

Gravity, electromagnetism, and the weak and strong interactions arise from the geometric curvature (rotations) in 6-dimensional field-space, without any separate fields or particles.

Explains unsolved phenomena:

- **Charge** as a particle-field projection $D_{14/24/34}$ of the electrically generated potential resulting from rotation relative to the dimensional plane D_{56}
- **SU(4)** symmetry dynamically breaks down to SU(3) × U(1), which leads to the weak interaction, where β geometrically describes the displacement relative to the dimensional plane D_{56}.
- **Dark energy** as a spin-0-pair of uncoupled photons with frequencies f less than the minimum coupling frequency f_{min}; after reaching fmin, decay into coupled invisible (dark) matter and visible matter
- **Avoiding** the **cosmological constant** (GTR) by $V(\phi_n)_{dark}$
- **Invisible (dark) matter** as compact wave-field oscillations with complex topological modes ($n \neq 0$, g $\geq$ 1) and Chern classes $c_3 = 0$
- **Expansion** from field deformation
- Resolution of the **wave-particle duality** through sinusoidal oscillations
- **Quantum entanglement:** Spin-0-pair state prior to decay
- **Fine-structure constant:** FSM: $\alpha = \frac{1}{136{,}6875}$ (3.33) ; Sommerfeld: $\alpha = \frac{1}{137{,}065}$
- **Three generations:** winding count n = 1, 2, 3 from the Fourier series
- **Anomaly-free:** Chern classes $c_1 = 0$, $c_3 = 0$

Scalability across orders:

The FSM model applies seamlessly from the Planck scale to the maximum extent of the universe. Hypothetically, it even applies to scales smaller than the Planck scale, once visible space disappears due to continued contraction and the universe reverts to the characteristics of a photon. Classically, GTR breaks down at the quantum level; QFT breaks down in the presence of gravity.

New particle model:

Point particles are replaced by 4-dimensional cavity modes with predictions for particle masses and coupling frequencies. Empirical data are confirmed on average with 99% agreement. The variances around the mean are smaller than the standard deviation of the measurement itself. Fions in the bundle generate charge and spin in the particle-field; exchange fions outside the bundle mediate interactions; passive fions form dark components. Spin, charge, and entanglement are purely rotational effects. The particle model is finite, geometric, and avoids singularities.

Practical Applications:

- **New propulsion** systems for space travel or near-Earth transport by leveraging the characteristics of a gravitational field at the physical level of conventional fields.
- **Optimized hot/cold fusion** with new heavy modes and stable fion clusters
- **Computers** with states of complete photon oscillations or spin-0-pair states for entangled qubits over macroscopic distances
- Production of **matter/antimatter-states** in stabilized forms
- **Gravitational wave detection** for predicting longitudinal and scalar modes in addition to the classical TT polarizations
- **Dark energy** technology for the targeted generation of $f > f_{min}$ for converting dark energy into usable photons

Added value compared to effective field theory in the QFT sense:

QFT is an effective theory involving renormalization, divergences, and arbitrary parameters. FSM is a fundamental geometric theory without renormalization. Masses, couplings, and charges arise directly from compactification and topology. The fine-structure constant and the three generations are not free parameters, but rather predictions. Quantum effects emerge from the dynamics of the compact wave-field dimensions (modes n/R, rotations), not from a formalism such as canonical quantization or path integrals. Thus, FSM avoids the infinities and renormalization that occur in QFT and solves problems such as duality geometrically (photons as waves in 4-dimensional subspaces).

- **Wave-particle duality:** Through sinusoidal rotations $V_{long} = c\cos(kt)$; $V_{trans} = c\sin(kt)$, the photon alternates between wave and particle states.
- **Discrete spectra:** From compactification (modes $\frac{n}{R}$; f_{Obj} according to (3.27))
- **Similar to the uncertainty principle:** Generated from periodic oscillation (kt), that produces position and momentum as averages over periods T.

- **No path integrals:** FSM does not use Feynman path integrals with sums over paths for probabilities; instead, photons follow null geodesics in the subspace ($ds^2 = 0$), which are classically determined but exhibit statistical behavior due to averaging over rotations.
- **Fundamental Theory:** FSM is not an effective field theory in the QFT sense (e.g., with cutoff and renormalization), nor does it have a Wilsonian EFT with path integrals. FSM is a fundamental theory that allows quantum effects to emerge, similar to condensed matter, where photons arise from classical lattice dynamics.
- **Planck's constant:** h is derived in FSM from geometric scales for $f_{Obj} \sim h\ f_e$. It avoids commutators and path integrals.
- **Quantum gravity:** FSM avoids problems such as those associated with quantum gravity and infinities by allowing quantum phenomena to emerge classically.

Advantages over string theory:

String theory requires 10 or 11 dimensions and supersymmetry for stability; strings are fundamental to it. It offers a unified framework that is hypothetical yet complex, without explicit predictions or testable parameters. The FSM model is potentially more powerful:

- **Simple dimensionality:** 7D instead of 10/11D, without supersymmetry, FSM stabilized by sine rotations
- **Explicit predictions:** Masses and frequencies can be calculated based on electrons. String theory, on the other hand, presents an infinite landscape without any predictions.
- **Geometric Matter:** Matter as a field deformation (no strings), scalable without additional assumptions. String theory posits the existence of strings, whereas FSM derives matter from relativistic space-time.
- **Explanation of dark phenomena:** Invisible/dark energy/matter arising from compact rotations; dualism is resolved, while string theory addresses a similar solution without, however, providing quantitative formulas.
- **Practical relevance:** FSM is testable and concrete, opening up entirely new applications. String theory, on the other hand, is abstract and difficult to prove empirically.

Added value compared to general relativity:

General Relativity explains gravity through geometry, but remains classical and cannot account for quantum effects, dark energy, and entanglement. FSM extends GTR to include a 7-dimensional geometry with compactified dimensions. Gravity remains a classical curvature, while quantum effects, particles, and dark energy arise from the same geometry.

Conclusion: FSM provides a unified, scalable geometric framework that resolves classical gaps from a single geometric foundation. It avoids the complexity of string theory and the renormalization problems of QFT, yet remains mathematically rigorous and experimentally testable. For verification, reference is made to the particle model, in which previously measured particles can be compared with calculated ones. **FSM is not merely a theory of particles and forces, but, in its full 7-dimensional structure, a theory of space-time itself.**